RÖ 2. SUBSTITUTIONSMETODEN, AREOR

A.5.6.8. Beräkna $\int x^2 2^{x^3+1}\,dx$.

Låt $t = x^3+1 \Rightarrow \frac{dt}{dx} = 3x^2 \Rightarrow dt = 3x^2\,dx$

$$\int x^2 2^{x^3+1}\,dx = \frac{1}{3}\int 2^{x^3+1}\underbrace{3x^2\,dx}_{dt} = \frac{1}{3}\int 2^t\,dt = \frac{1}{3}\int e^{t\cdot\ln 2}\,dt =$$

$$= \frac{1}{3\ln 2}e^{t\ln 2} + C = \frac{1}{\ln 8}2^{x^3+1} + C$$

A.5.6.14. Beräkna $\int \frac{x+1}{\sqrt{x^2+2x+3}}\,dx$

Låt $t = x^2+2x+3 \Rightarrow dt = (2x+2)\,dx$

$$\int \frac{x+1}{\sqrt{x^2+2x+3}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt = \sqrt{t} + C = \sqrt{x^2+2x+3} + C$$

A.5.6.38. Beräkna $\int \frac{\cos^4 x}{\sin^8 x}\,dx$.

$$\int \frac{\cos^4 x}{\sin^8 x}\,dx = \int \frac{\cos^4 x}{\sin^4 x}\cdot\frac{1}{\sin^4 x}\,dx = \int \cot^4 x \cdot \frac{1}{\sin^4 x}\,dx =$$

$$= \left\{ \sin^2 x = 1-\cos^2 x = 1 - \frac{\cos^2 x}{\sin^2 x}\cdot\sin^2 x = 1 - \cot^2 x\cdot\sin^2 x \right.$$

$$\Rightarrow \sin^2 x(1+\cot^2 x) = 1 \Rightarrow \frac{1}{\sin^2 x} = (1+\cot^2 x)$$

$$\left. \text{Låt } t = \cot x \Rightarrow dt = \frac{d}{dx}\cot x\,dx = -\frac{1}{\sin^2 x}\,dx \right\} =$$

$$= \int \cot^4 x(1+\cot^2 x)\cdot\frac{1}{\sin^2 x}\,dx = \longrightarrow$$

↑ behåll en term $1/\sin^2 x$ som inre derivata!

$= \int t^4(1+t^2)\cdot(-1)\,dt = -\int t^4 + t^6\,dt = -\frac{t^5}{5} - \frac{t^7}{7} + C$

$= -\frac{\cot^5 x}{5} - \frac{\cot^7 x}{7} + C$

Stämmer detta?

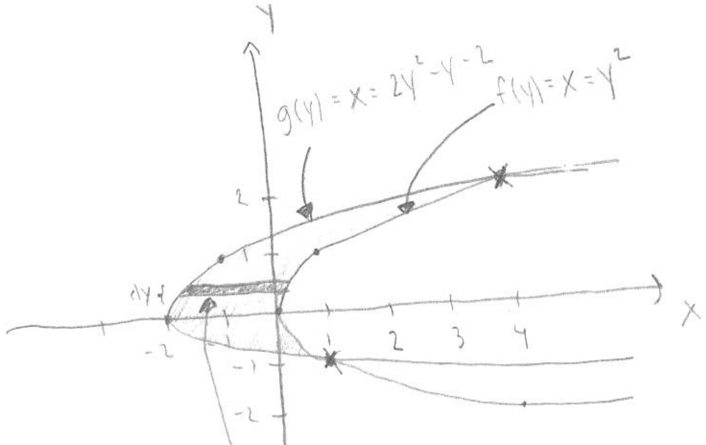
$\frac{d}{dx}\left(-\frac{\cot^5 x}{5} - \frac{\cot^7 x}{7}\right) = -\frac{5\cot^4 x}{5}\cdot\left(-\frac{1}{\sin^2 x}\right) - \frac{7\cot^6 x}{7}\cdot\left(-\frac{1}{\sin^2 x}\right)$

$= \frac{\cos^4 x}{\sin^6 x} + \frac{\cos^6 x}{\sin^8 x} = \{\text{gemensam nämnare}\} =$

$= \frac{\cos^4 x \sin^2 x + \cos^6 x}{\sin^8 x} = \frac{\cos^4 x}{\sin^8 x}(\underbrace{\sin^2 x + \cos^2 x}_{=1}) = \frac{\cos^4 x}{\sin^8 x}$ ok!

A.5.7.10 Skissa och beräkna arean av området som begränsas av kurvorna $x = y^2$ och $x = 2y^2 - y - 2$.

Funktioner av y! Gör på samma sätt som vanligt, men areaelementen är nu horisontella istället.



skärningspunkter:

$y^2 = 2y^2 - y - 2 \iff$

$y^2 - y - 2 = 0 \iff$

$(y - \frac{1}{2})^2 = 2 + \frac{1}{4} = \frac{9}{4} \iff$

$y = \frac{1}{2} \pm \frac{3}{2} \Rightarrow y_1 = 2,\ y_2 = -1$

$A = \int_{y=-1}^{2} |\underbrace{f(y) - g(y)}| \, dy = \{f(y) \geq g(y)$ på $[-1, 2] \Rightarrow$ ta bort beloppet$\}$

$$= \int_{-1}^{2} y^2 - 2y^2 + y + 2 \, dy = \int_{-1}^{2} -y^2 + y + 2 \, dy = \left[-\frac{y^3}{3} + \frac{y^2}{2} + 2y \right]_{-1}^{2}$$

$$= -\frac{8}{3} + \frac{4}{2} + 4 - \left(\frac{1}{3} + \frac{1}{2} - 2 \right) = -\frac{9}{3} + \frac{3}{2} + 6 = \frac{-18 + 9 + 36}{6} =$$

$$= \frac{27}{6} = \frac{9}{2} = 4.5 \text{ a.e.}$$