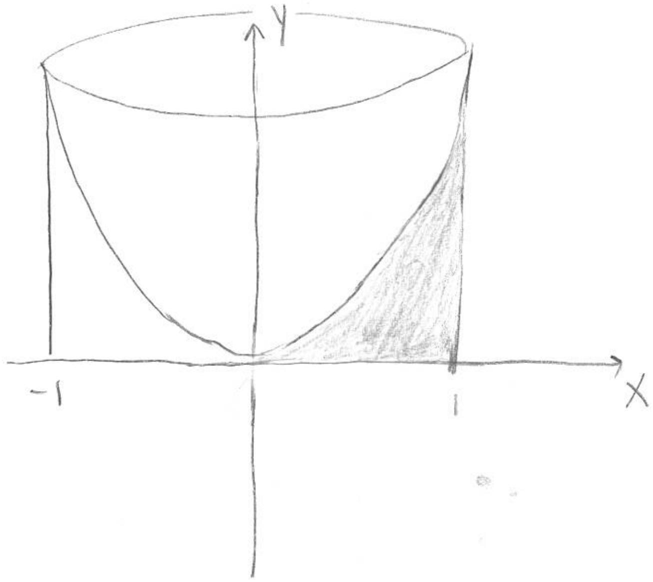
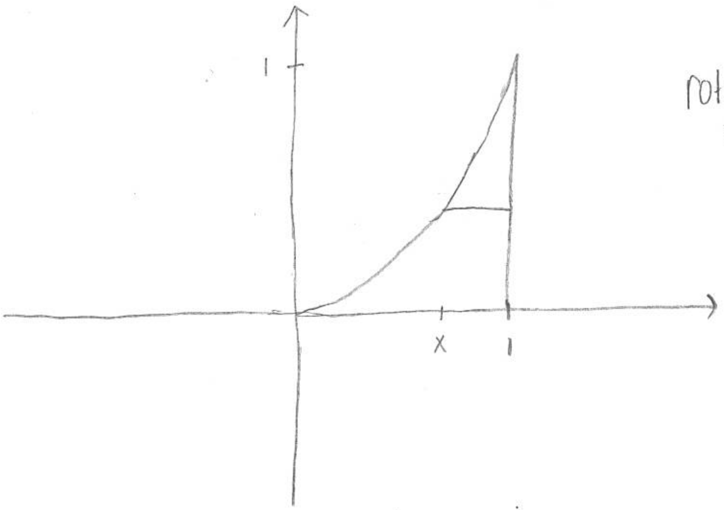
RÖ 5. ROTATIONSKROPPAR

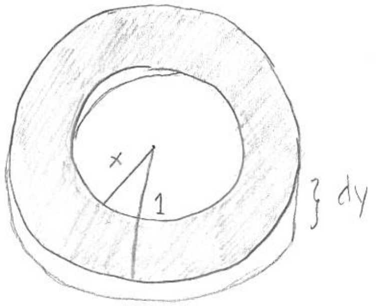
A.7.1.2. Beräkna volymen av rotationskroppen som uppstår då området som begränsas av linjerna $y = x^2$, $y = 0$ och $x = 1$ roteras runt y-axeln.



Cirkelskivor



roterar runt y-axeln $\Rightarrow$



Arean av cirkelskivan

$$A(x) = \pi(1^2 - x^2) = \pi(1 - x^2)$$

$\Rightarrow A(y) = \pi(1 - y)$ eftersom $x^2 = y$.

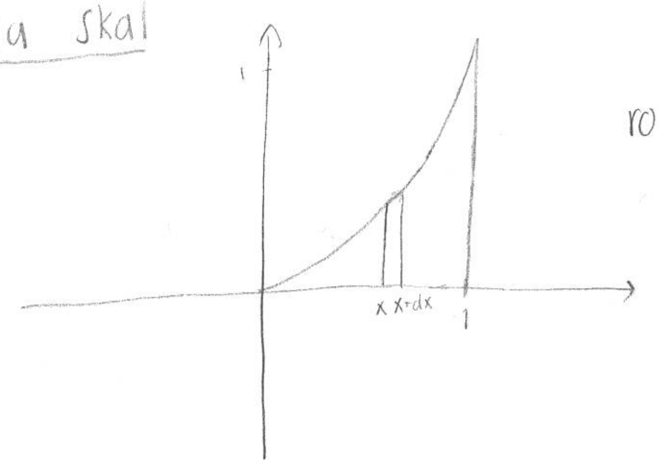
summera upp alla små volymselement.

$$V = \int_V dv = \int_{y=0}^{1} A(y)\,dy$$

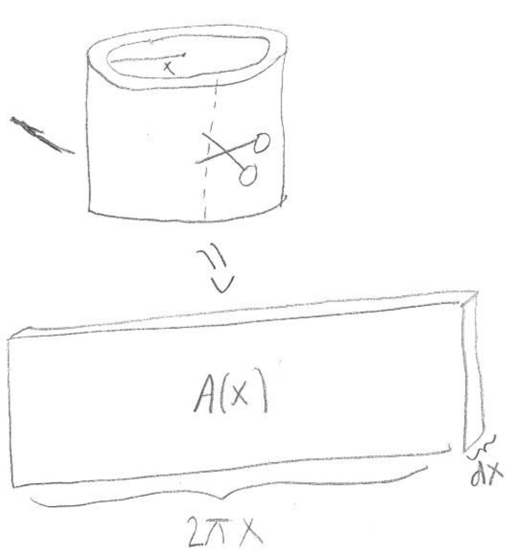
$$= \pi \int_0^1 (1-y)\,dy = \pi \left[y - \frac{y^2}{2} \right]_0^1 = \pi\left(1 - \frac{1}{2}\right) = \frac{\pi}{2} \text{ v.e.}$$

vi vill summera upp areaelementen i y-led eftersom cirkelskivorna breder ut sig i x-led.

Cylindriska skal



rotera runt
y-axeln
⇒

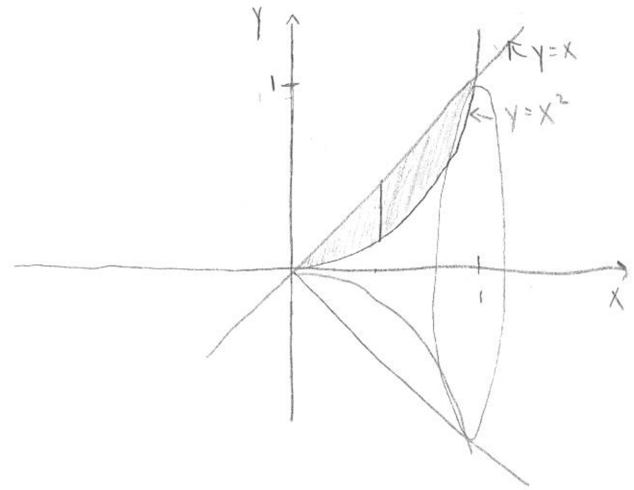


$$V = \int_{x=0}^{1} A(x)\,dx = \qquad y = x^2$$

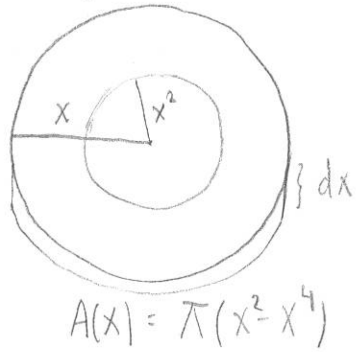
$$= 2\pi \int_0^1 x^3\,dx = 2\pi \left[\frac{x^4}{4}\right]_0^1 = 2\pi \cdot \frac{1}{4} = \frac{\pi}{2} \text{ v.e.}$$

A.7.1.6. Beräkna volymen av objektet som uppstår då området som begränsas av kurvorna $y = x$ och $y = x^2$ roteras runt
a) x-axeln, b) y-axeln.

a)



rotera runt
x
⇒



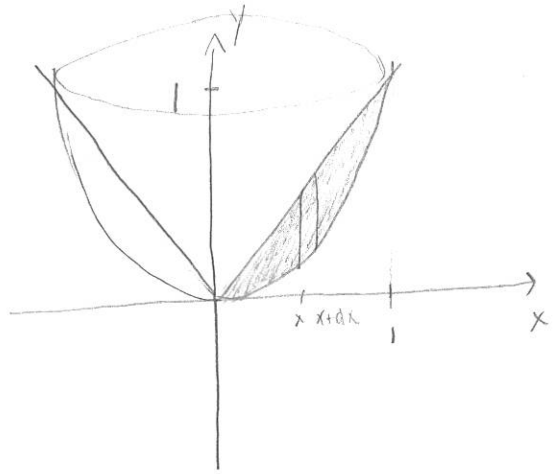
$A(x) = \pi(x^2 - x^4)$

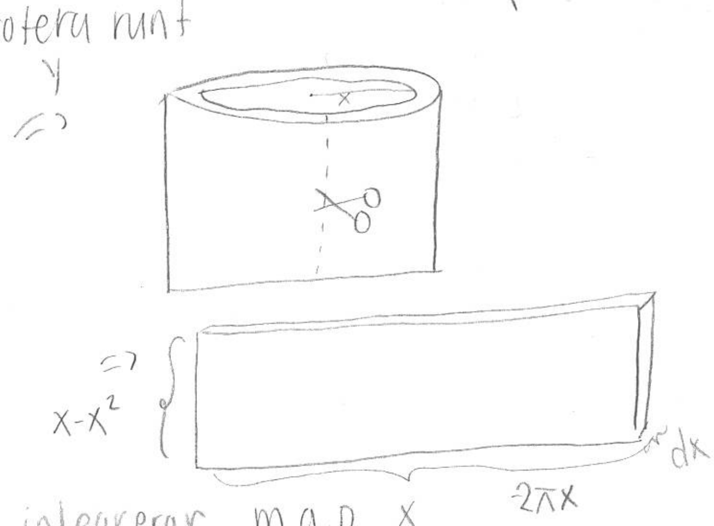
Vi får cirkelskivor om vi integrerar m.a.p. x.

$$V = \int_{x=0}^{1} A(x)\,dx = \pi \int_0^1 (x^2 - x^4)\,dx = \pi \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \pi\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$= \frac{2\pi}{15} \text{ v.e.}$$

b)



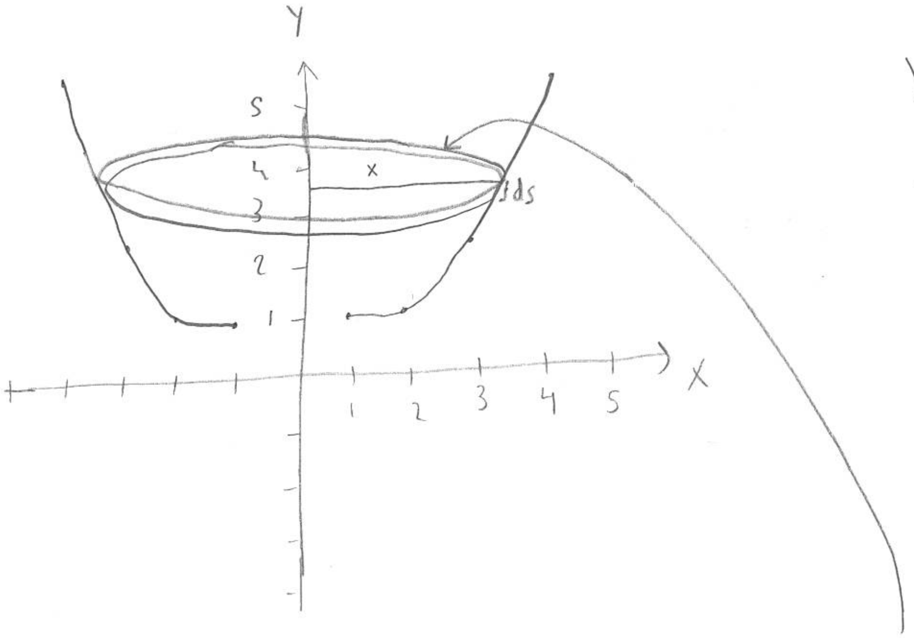


Vi får cylindriska skal om vi integrerar m.a.p. x.

$$V = 2\pi \int_{x=0}^{1} x(x-x^2)\,dx = 2\pi\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 2\pi\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{\pi}{6} \text{ v.e.}$$

A.7.3.27. Beräkna arean av den yta som uppstår när kurvan

$$y = \frac{x^3}{12} + \frac{1}{x}, \quad 1 \le x \le 4, \text{ roteras runt y-axeln}$$



$$y(1) = \frac{1}{12} + 1 = \frac{13}{12} \approx 1$$

$$y(2) = \frac{8}{12} + \frac{1}{2} = \frac{8+6}{12} = \frac{7}{6} \approx 1$$

$$y(3) = \frac{27}{12} + \frac{1}{3} = \frac{27+4}{12} = \frac{31}{12} \approx 2.5$$

$$y(4) = \frac{64}{12} + \frac{1}{4} = \frac{67}{12} \approx 5.5$$

$dS = 2\pi x\, ds$ är arean av det cirkulära bandet.

Här är ds båglängdselement

$$ds = \sqrt{1 + (f'(x))^2}\,dx = \sqrt{1 + (y'(x))^2}\,dx = \sqrt{1 + \left(\frac{x^2}{4} - \frac{1}{x^2}\right)^2}\,dx =$$

$$\sqrt{1+\frac{x^4}{16}-\frac{1}{2}+\frac{1}{x^4}}\,dx = \frac{1}{4}\sqrt{x^4-8+\frac{16}{x^4}}\,dx = \frac{1}{4}\sqrt{\left(x^2-\frac{4}{x^2}\right)^2}\,dx$$

$$= \frac{1}{4}\left(x^2-\frac{4}{x^2}\right)dx$$

Så $dS = 2\pi x\,ds = \frac{2\pi x}{4}\left(x^2-\frac{4}{x^2}\right)dx$

Arean får vi om vi integrerar upp alla areaelement dS.

$$A = \int_{x=1}^{4} dS = \frac{\pi}{2}\int_1^4 x\left(x^2-\frac{4}{x^2}\right)dx = \frac{\pi}{2}\left[\frac{x^4}{4}+4\ln x\right]_1^4$$

$$= \frac{\pi}{2}\left(64+4\ln 4-\frac{1}{4}\right) = \frac{\pi}{8}\left(255+32\ln 4\right) \text{ a.e.}$$