Lösningar till tentan i LGMA40/L9MA45

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1) $(x+1)y' + 2y = (x+1)^4$

$$y' + \frac{2}{x+1}y = (x+1)^3$$

Hitta integrerande faktor: $\int \frac{2}{x+1}\,dx = 2\ln|x+1|$

$e^{2\ln|x+1|} = e^{\ln(x+1)^2} = (x+1)^2$. Multiplicera med detta:

$$(x+1)^2 y' + 2(x+1)y = (x+1)^5$$

$$\left((x+1)^2 y\right)' = (x+1)^5$$

$$(x+1)^2 y = \frac{(x+1)^6}{6} + C$$

$$y = \frac{(x+1)^4}{6} + \frac{C}{(x+1)^2}$$

$y(0)=0$ ger: $0 = \frac{1}{6} + C \Rightarrow C = -\frac{1}{6}$

Svar: $y = \frac{(x+1)^4}{6} - \frac{1}{6(x+1)^2}$

2. $y' = -y^2 x \sin(x^2+4)$ $\qquad y' = \frac{dy}{dx}$

$$-\frac{dy}{y^2} = x\sin(x^2+4)\,dx$$

$$-\int \frac{dy}{y^2} = \int x\sin(x^2+4)\,dx$$

$$\frac{1}{y} = \int x\sin(x^2+4)\,dx = \left\{\begin{array}{l} t = x^2+4 \\ dt = 2x\,dx \end{array}\right\} =$$

$$= \frac{1}{2}\int \sin(t)\,dt = -\frac{1}{2}\cos(t) + C =$$ (2)

$$= -\frac{1}{2}\cos(x^2+4) + C = \frac{1}{y}.$$

Så $y = -\dfrac{2}{\cos(x^2+4)+C}$

(3.) Låt $y(t)$ = befolkningsmängden år t.
Varje år ökar befolkningen med
$0{,}002 \cdot y(t) + 700 - 200$, så

$y' = 0{,}002 \cdot y + 500$ och $y(0) = 200\,000$.

(4.) (a.) $\dfrac{3^k}{2^{2k}} = \dfrac{3^k}{(2^2)^k} = \dfrac{3^k}{4^k} = \left(\dfrac{3}{4}\right)^k$, så

$$\sum_{k=1}^{\infty} \frac{3^k}{2^{2k}} = \sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k = \frac{3}{4}\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^{k-1} =$$

$$= \frac{3}{4}\sum_{k=0}^{\infty}\left(\frac{3}{4}\right)^k = \frac{3}{4}\,\frac{1}{1-\frac{3}{4}} = \frac{3}{4-3} = 3$$

konvergent!

(b.) $\dfrac{e^k}{e^k - k} = \dfrac{1}{1-\frac{k}{e^k}} \to 1$ när $k \to \infty$.

$\frac{k}{e^k} \to 0$ när $k \to \infty$

Eftersom seriens termer inte går mot noll, så är serien divergent.

(5.) $f(x) = \ln(3-2x)$ $\qquad f(1) = \ln(1) = 0$

$f'(x) = -2 \cdot \frac{1}{3-2x}$ $\qquad f'(1) = -2$ $\qquad$ (3)

$f''(x) = -(-2)^2 \frac{1}{(3-2x)^2}$ $\qquad f''(1) = -4$

$f'''(x) = 2 \cdot (-2)^3 \frac{1}{(3-2x)^3}$

Svar: $p_2(x) = -2(x-1) - \frac{4(x-1)^2}{2}$

med restterm $R_3(x) = -8 \frac{1}{(3-2\xi)^3} \cdot \frac{(x-1)^3}{6}$

där ξ ligger mellan x och 1.

(6.) $\arctan(x) = x - \frac{x^3}{5} + B_1(x)x^5$

$\sin(x) = x - \frac{x^3}{6} + B_2(x)x^5$

$\cos(x) = 1 - \frac{x^2}{2} + B_3(x)x^4$

$\cos(x^2) = 1 - \frac{x^4}{2} + B_4(x)x^8$

$$\frac{\arctan(x) - \sin(x)}{\cos(x^2) - 1} = \frac{-\frac{x^3}{5} + \frac{x^3}{6} + B_5(x)x^5}{-\frac{x^4}{2} + B_4(x)x^8} =$$

$$= \frac{\left(-\frac{6}{30} + \frac{5}{30}\right) \cdot x^3 + B_5(x)x^5}{-\frac{x^4}{2} + B_4(x)x^8} = \frac{-\frac{x^3}{30} + B_5(x)x^5}{-\frac{x^4}{2} + B_4(x)x^8} =$$

$$= \frac{\frac{1}{30} - B_5(x)x}{\frac{x}{2} - B_4(x)x^4}$$

Gränsvärdet av detta när $x \to 0$ existerar inte eftersom täljaren $\to \frac{1}{30}$ och nämnaren $\to 0$.

7. $4y^2+9x^2-8y+18x+12=0$

$4(y^2-2y)+9(x^2+2x)+12=0$

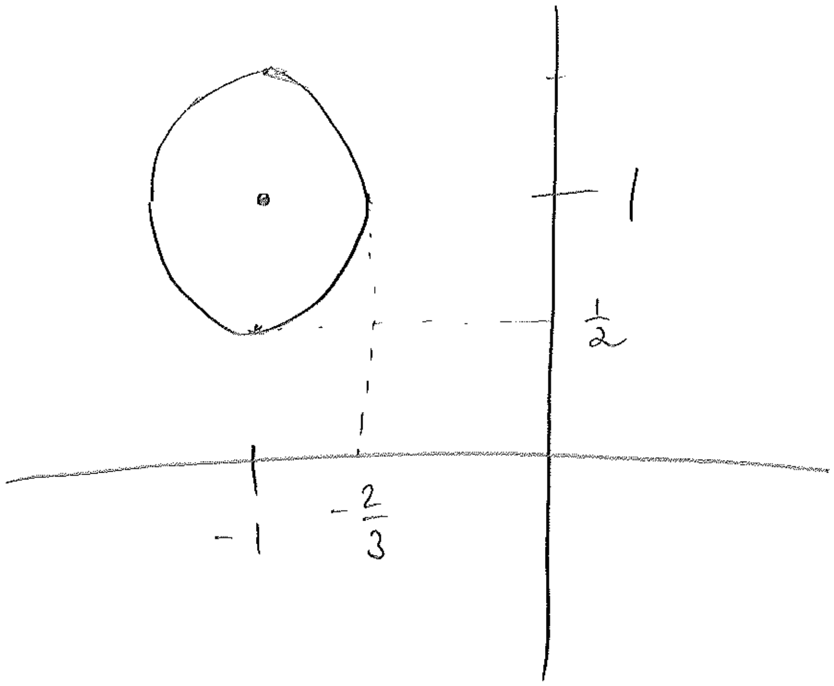
$4(y-1)^2-4+9(x+1)^2-9+12=0$

$4(y-1)^2+9(x+1)^2=1$

$$\frac{(y-1)^2}{\left(\frac{1}{2}\right)^2}+\frac{(x+1)^2}{\left(\frac{1}{3}\right)^2}=1.$$

Detta är en ellips med centrum i $(-1,1)$

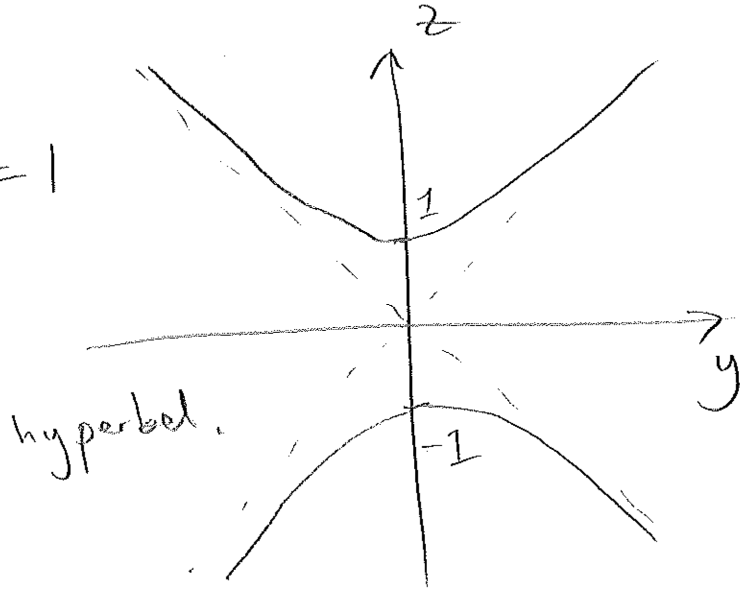
Halvaxlarna är $\frac{1}{3}$ och $\frac{1}{2}$:



8. $4x^2+z^2=1+y^2$

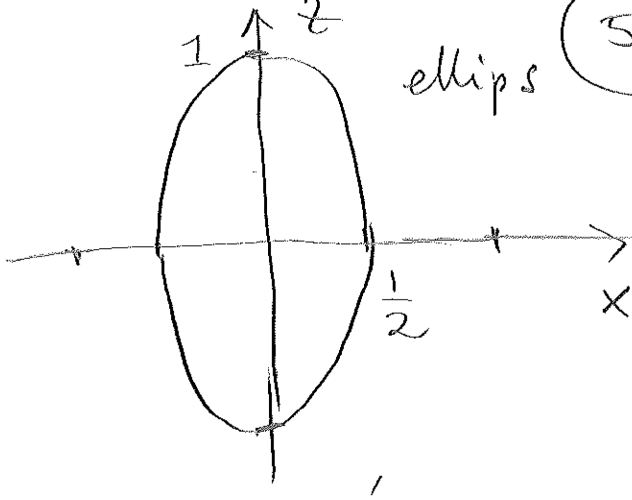
Sätt $x=0$: $z^2-y^2=1$

Ger skärningen med yz-planet.



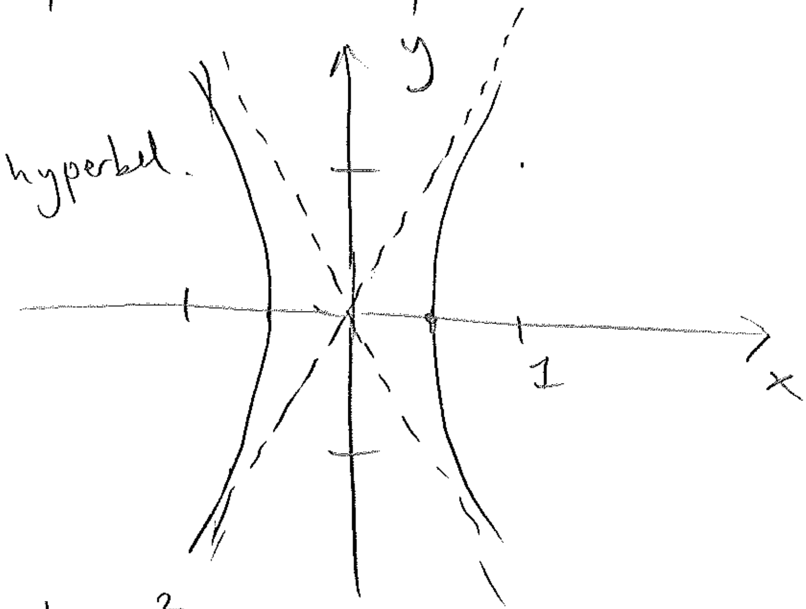
Sätt $y = 0$: $4x^2 + z^2 = 1$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2} + z^2 = 1$$



Sätt $z = 0$: $4x^2 - y^2 = 1$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2} - y^2 = 1$$

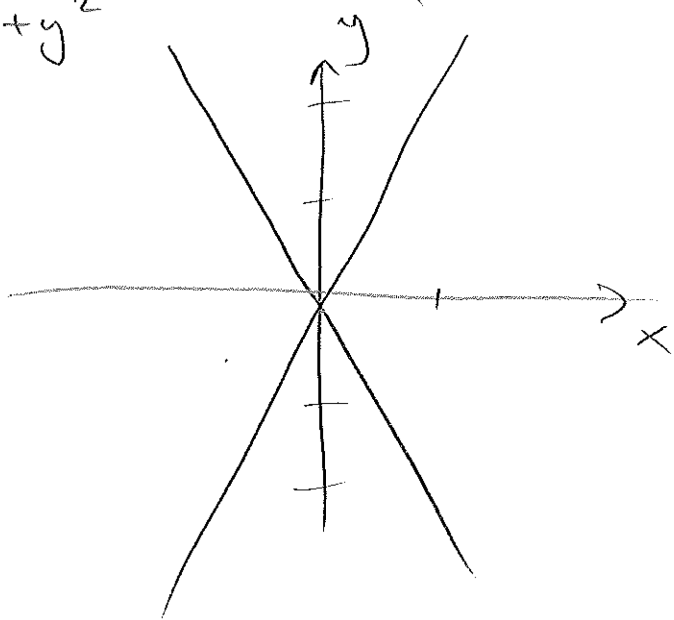


asymptotenas lutning: $\frac{1}{\frac{1}{2}} = 2$

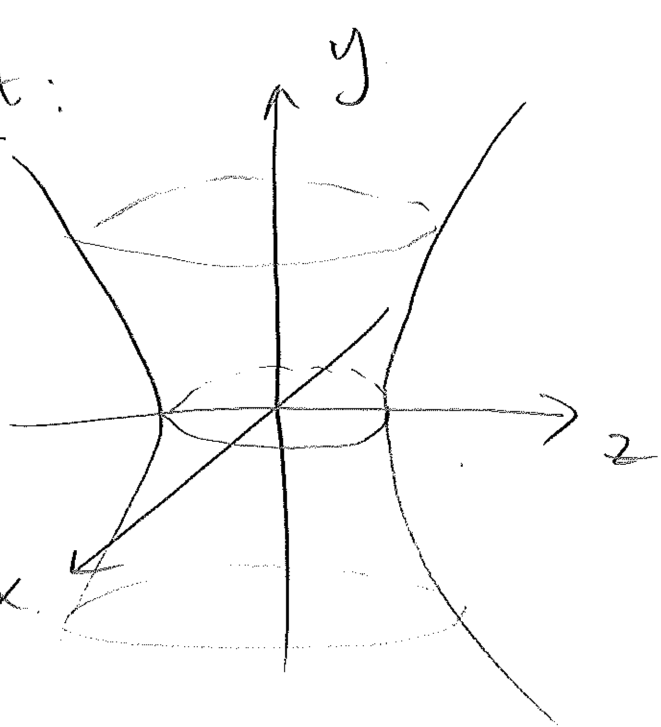
Nivåkurvorna till $z = \pm 1$: $4x^2 + 1 = 1 + y^2$

$$4x^2 - y^2 = 0$$

$$y = \pm 2x$$



Totalt:



Detta är en enmantlad hyperboloid.