FACIT till dugga 20111101

1. $E[x] = -\frac{1}{2} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{3} = -\frac{1}{3} + \frac{2}{9} = \frac{2-3}{9} = \boxed{-\frac{1}{9}}$

2. $V(x) = E[x^2] - E[x]^2 \Rightarrow E[x] = \pm\sqrt{E[x^2] - V(x)} = \pm 3$
   men $E[x] \geq 0$ så $\underline{E[x] = 3.}$

3. Disjunkta $\Rightarrow A \cap B = \emptyset$; men $B^c = A$ ty $A \cup B = \Omega$ $(P(A) + P(B) = 1$ och
   disjunkta) $\Rightarrow P(A \cap B) = \underline{0}$ och $P(A \cap B^c) = P(A) = \underline{0.2}$

4. Oberoende $\Rightarrow P(A \cap B) = P(A)P(B) = 0.2 \cdot 0.8 = \underline{0.16}$
   $P(A \cap B^c) = P(A)P(B^c) = P(A)(1 - P(B)) = 0.2 \cdot 0.2 = \underline{0.04}$

5. $P(\text{rättsvar}) = P(\text{rätt svar} | \text{frisk})P(\text{frisk}) + P(\text{rätt svar} | \text{sjuk})P(\text{sjuk})$
   $= \cancel{0.15 \cdot \frac{9}{10} + 0.85 \cdot \frac{1}{1}}$

   $= 0.99 \cdot \frac{9}{10} + 0.95 \cdot \frac{1}{10} = 0.986 \Rightarrow \boxed{98.6\%}$

6. $P(\text{miss}) = 0.1 \Rightarrow P(\text{rätt}) = 0.9 = 1 - p$
   geometrisk fördelning (oberoende kast): $X \sim Geo(1-p)$ till
   första rätt. Så $E[X] = \frac{1}{1-p} = \frac{1}{0.9} = \boxed{1.11}$

7. $E\left[\frac{x - \mu}{b}\right] = \frac{1}{b}(E[x] - \mu) = \frac{1}{b}(\mu - \mu) = 0$

   $\cancel{V[x]} = V\left(\frac{x - \mu}{b}\right) = \frac{1}{b^2}V(x) = \frac{1}{b^2} \cdot b^2 = 1$

   $\Rightarrow \boxed{\frac{x - \mu}{b} \sim N(0,1)}$