MAL600 Flervariabelanalys 4 juni 04

1. $(2x, 2y, 2z)_{(1,-1,2)} = 2(1,-1,2)$

$(6x^2+6xy^2-4y,\ 6x^2y+3y^2-4x,\ -3z^2)_{(1,-1,2)} = (16,-7,-12)$

$\begin{pmatrix}1\\-1\\2\end{pmatrix} \times \begin{pmatrix}16\\-7\\-12\end{pmatrix} = \begin{pmatrix}26\\44\\9\end{pmatrix}$ så tangentens ekv. är $\begin{cases} x = 1+26t \\ y = -1+44t \\ z = 2+9t \end{cases}$

2. Med $x = 2u-5v$, $y = 3v-w$ gäller $z'_u = z'_x x'_u + z'_y y'_u = 2z'_x$,
$z'_v = z'_x x'_v + z'_y y'_v = -5z'_x + 3z'_y$ och $z'_w = z'_x x'_w + z'_y y'_w = -z'_y$
Så $5z'_u + 2z'_v + 6z'_w = (5\cdot 2 + 2(-5))z'_x + (2\cdot 3 + 6\cdot(-1))z'_y = 0$.

3. $\begin{cases} f'_x = 3x^2+3y^2-3 = 0 \Leftrightarrow x^2+y^2=1 \\ f'_y = 6xy-6y^2 = 0 \Leftrightarrow (x-y)y=0 \end{cases} \Leftrightarrow$ $y=0,\ x=\pm 1$ el. $x=y=\pm\frac{1}{\sqrt2}$

	(x,y)	$(1,0)$	$(-1,0)$	$(\frac{1}{\sqrt2},\frac{1}{\sqrt2})$	$(-\frac{1}{\sqrt2},-\frac{1}{\sqrt2})$
$A = f''_{xx}$	$6x$	$6>0$	$-6<0$	$3\sqrt2$	$-3\sqrt2$
$B = f''_{xy}$	$6y$	0	0	$3\sqrt2$	$-3\sqrt2$
$C = f''_{yy}$	$6x-12y$	6	-6	$-3\sqrt2$	$3\sqrt2$
$AC-B^2$		$36>0$	$36>0$	$-36<0$	$-36<0$
		lok. min. ($f=-2$)	lok. max. ($f=2$)	Sadelpunkter ($f=-\sqrt2$)	($f=\sqrt2$)

4. $\begin{cases} yz = 3\lambda x^2 \\ xz = 3\lambda y^2 \\ xy = 3\lambda z^2 \end{cases}$ $\Rightarrow xyz = 3\lambda x^3 = 3\lambda y^3 = 3\lambda z^3$

$x^3+y^3+z^3=1$
$x, y, z \geq 0$

I. $\lambda=0 \Rightarrow yz = xz = xy = 0 \Rightarrow$ två av x,y,z är 0 ($\Rightarrow$ den tredje är 1) $\Rightarrow f=0$ = min

II. $x=y=z \Rightarrow x=y=z=\frac{1}{\sqrt[3]{3}} \Rightarrow f=\frac13$ = max

5. $\int_0^{\sqrt3} \frac{1}{x^2}\left(\int_0^{x^2} \frac{1}{1+y}\,dy\right)dx = \int_0^{\sqrt3} \frac{1}{x^2}\Big[\ln(1+y)\Big]_0^{x^2}dx = \int_0^{\sqrt3}\frac{1}{x^2}\ln(1+x^2)\,dx =$

$= \left[-\frac1x \ln(1+x^2)\right]_0^{\sqrt3} + \int_0^{\sqrt3} \frac1x \cdot 2x \cdot \frac{1}{1+x^2}\,dx = -\frac{1}{\sqrt3}\ln 4 + \lim_{x\to0+}\frac{\ln(1+x^2)}{x} +$

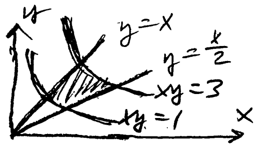
$+ \Big[2\arctan x\Big]_0^{\sqrt3} = -\frac{2}{\sqrt3}\ln 2 + \lim_{x\to0+} \underbrace{x}_{\to 0}\cdot\underbrace{\frac{\ln(1+x^2)}{x^2}}_{\to 1} + 2\cdot\frac{\pi}{3} = \frac{2\pi}{3} - \frac{2\ln 2}{\sqrt3}$

eller

$\int_0^3 \frac{1}{1+y}\left(\int_{\sqrt y}^{\sqrt3}\frac{1}{x^2}\,dx\right)dy = \int_0^3 \frac{1}{1+y}\left[-\frac1x\right]_{\sqrt y}^{\sqrt3}dy = \int_0^3\left(\frac{1}{\sqrt y(1+y)} - \frac{1}{\sqrt3}\cdot\frac{1}{1+y}\right)dy =$

$= \left[2\arctan\sqrt y - \frac{1}{\sqrt3}\ln(1+y)\right]_0^3 = 2\cdot\frac{\pi}{3} - \frac{1}{\sqrt3}\ln 4 = \frac{2\pi}{3} - \frac{2\ln 2}{\sqrt3}$

6. $y \le x \le 2y \Rightarrow y \le 2y \Rightarrow y \ge 0 \Rightarrow x \ge 0$;



$1 \le xy$ medför sedan $y > 0$ och $x > 0$ så

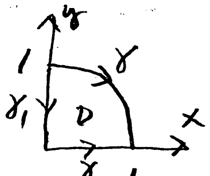
D kan skrivas $\quad 1 \le \frac{x}{y} \le 2, \; 1 \le xy \le 3, \; x, y > 0.$

Vi prövar därför substitutionen

$$\begin{cases} u = \frac{x}{y} \\ v = xy \end{cases} \Rightarrow \frac{d(u,v)}{d(x,y)} = \begin{vmatrix} \frac{1}{y} & -\frac{x}{y^2} \\ y & x \end{vmatrix} = \frac{x}{y} + \frac{x}{y} = u + u = 2u$$

och $\iint_D x^2 e^{\frac{x}{y}} dx\,dy = \int_1^2 \left(\int_1^3 uv\, e^u \frac{1}{2u}\, dv \right) du = \frac{1}{2} \left[\frac{v^2}{2} \right]_1^3 \left[e^u \right]_1^2 =$

$= \frac{1}{4}(9-1)(e^2 - e) = 2(e^2 - e)$

7. 

$\int_{\gamma_1 + \gamma_2 - \gamma} = \iint_D$, så

$\int_\gamma (x + y^3)dx + (6x+1)y^2 dy = \int_{\gamma_1} (0 + y^2) dy + \int_{\gamma_2} x\,dx -$

$- \iint_D (6y^2 - 3y^2) dx\,dy = \int_1^0 y^2 dy + \int_0^1 x\,dx - \int_0^1 \left(\int_0^{\sqrt[3]{1-x^3}} 3y^2 dy \right) dx =$

$= \left[\frac{y^3}{3} \right]_1^0 + \left[\frac{x^2}{2} \right]_0^1 - \int_0^1 \left[y^3 \right]_0^{\sqrt[3]{1-x^3}} dx = -\frac{1}{3} + \frac{1}{2} - \int_0^1 (1 - x^3) dx =$

$= \frac{1}{6} - 1 + \frac{1}{4} = -\frac{7}{12}$

8. a) I polära koordinater är

$f = \frac{r^3 \cos^3\theta}{r^2} = r\cos^3\theta$ som $\to 0$ då $r \to 0$.

b) $f(x,0) = x$ så $f'_x(x,0) = 1$ för alla x (även $x = 0$).

$f(0,y) = 0$ så $f'_y(0,y) = 0$ för alla y (även $y = 0$).

$$\frac{f(x,y) - f(0,0) - f'_x(0,0)x - f'_y(0,0)y}{\sqrt{x^2+y^2}} = \frac{x^3 - x^3 - xy^2}{(x^2+y^2)^{3/2}} =$$

$$= -\frac{xy^2}{(x^2+y^2)^{3/2}} = \left(\begin{matrix} \text{pol.} \\ \text{koord.} \end{matrix} \right) = -\cos\theta \sin^2\theta \not\to 0, \; r \to 0.$$

Alltså: Svar a) Ja b) Nej.