LMA320/MAL600 Flervariabelanalys 18 aug 05

**1.** $\vec{r}(s,t)=\begin{pmatrix}s^2\\st\\t^2\end{pmatrix}=\begin{pmatrix}4\\6\\9\end{pmatrix} \Leftrightarrow s=2, t=3 \quad (s,t>0)$

$\vec{r}\,'_s=\begin{pmatrix}2s\\t\\0\end{pmatrix}=\begin{pmatrix}4\\3\\0\end{pmatrix}, \quad \vec{r}\,'_t=\begin{pmatrix}0\\s\\2t\end{pmatrix}=\begin{pmatrix}0\\2\\6\end{pmatrix}$ i punkten

så tangentplanet kan skrivas $\begin{cases}x=4+4s\\y=6+3s+2t\\z=9\qquad+6t\end{cases}$ $s,t\in R$

eller använd $\vec{r}\,'_s\times\vec{r}\,'_t=(18,-24,8)=2(9,-12,4)$ så ekv. är $0=9(x-4)-12(y-6)+4(z-9)=9x-12y+4z.$

**2.** $\begin{cases}u=x^2-y^2\\v=xy\end{cases}$ $\quad z'_x=z'_u u'_x+z'_v v'_x=2xz'_u+yz'_v$
$\quad z'_y=z'_u u'_y+z'_v v'_y=-2yz'_u+xz'_v$

så $xz'_x-yz'_y=2(x^2+y^2)z'_u$ som är $=x^2+y^2 \Leftrightarrow z'_u=\frac{1}{2}$

$\Leftrightarrow z=\frac{1}{2}u+f(v)=\frac{1}{2}(x^2-y^2)+f(xy)$, $f$ godt. deriverbar fkn

b) $0=z(x,1)=\frac{1}{2}(x^2-1)+f(x) \Rightarrow f(x)=\frac{1}{2}(1-x^2)$ så
$z=\frac{1}{2}(x^2-y^2)+\frac{1}{2}(1-(xy)^2)=\frac{1}{2}(1+x^2-y^2-x^2y^2)$

**3.** $f(x,y)=(x^2-3y^2)e^{-x^2-y^2}$, $f(x,y)\to 0$ då $x^2+y^2\to\infty$.

$f'_x=2xe^{-x^2-y^2}-2x(x^2-3y^2)e^{-x^2-y^2}=2x(1-x^2+3y^2)e^{-x^2-y^2}$

$f'_y=-6ye^{-x^2-y^2}-2y(x^2-3y^2)e^{-x^2-y^2}=-2y(3+x^2-3y^2)e^{-x^2-y^2}$

$f'_x=0 \Leftrightarrow x=0$ (1) el. $x^2-3y^2=1$ (2) $\quad f'_y=0\Leftrightarrow y=0$ (3) el. $x^2-3y^2=-3$ (4)

(1)&(3) ger $(0,0)$; $f(0,0)=0$ $\quad$ (1)&(4) ger $(0,\pm1)$; $f(0,\pm1)=-3/e$

(2)&(3) ger $(\pm1,0)$; $f(\pm1,0)=1/e$ $\quad$ (2)&(4) går inte.

Alltså $f_{max}=\frac{1}{e}=f(\pm1,0)$ $\quad f_{min}=-3/e=f(0,\pm1)$.

**4.** Max och min av $f(x,y)=x^2+y^2$ då $g(x,y)=x^4+x^2y^2+2y^4=1$. (B)

$\nabla g=(4x^3+2xy^2, 2x^2y+8y^3)$ $\quad \nabla g\neq\vec{0}$ då $g=1$. $\quad \nabla f=(2x,2y)$.

$\nabla f /\!/ \nabla g \Leftrightarrow 0=\begin{vmatrix}x & y\\2x^3+xy^2 & x^2y+4y^3\end{vmatrix}=xy(x^2+4y^2)-xy(2x^2+y^2)=xy(3y^2-x^2)$

1. $x=0 \overset{(B)}{\Rightarrow} 2y^4=1 \Rightarrow y^2=\frac{1}{\sqrt2} \Rightarrow f=\frac{1}{\sqrt2}=f_{min}$
2. $y=0 \Rightarrow x^4=1 \Rightarrow x^2=1 \Rightarrow f=1$
3. $x^2=3y^2 \Rightarrow 9y^4+3y^4+2y^4=1 \Rightarrow y^2=\frac{1}{\sqrt{14}} \Rightarrow x^2=\frac{3}{\sqrt{14}} \Rightarrow f=\frac{4}{\sqrt{14}}=f_{max}$

**5.** $D:\ 1<xy<3,\ 2<\frac{y}{x}<4,\ x,y>0$

$$\begin{cases} u=xy \\ v=\frac{y}{x}\end{cases} \Rightarrow \frac{d(u,v)}{d(x,y)} = \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x}\end{vmatrix} = 2\frac{y}{x} = 2v$$

$$\iint_D x^2\,dx\,dy = \int_1^3 du \int_2^4 \frac{u}{v}\cdot\frac{1}{2v}\,dv = \frac{1}{2}\int_1^3 u\,du \int_2^4 \frac{1}{v^2}\,dv =$$

$$= \frac{1}{2}\left[\frac{1}{2}u^2\right]_1^3 \left[-\frac{1}{v}\right]_2^4 = \frac{1}{2}\cdot\frac{1}{2}\cdot(9-1)\cdot\left(\frac{1}{2}-\frac{1}{4}\right) = \frac{1}{2}.$$

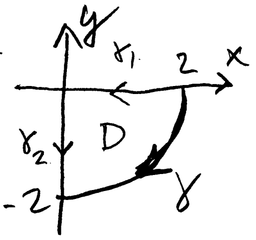
**6.** $D:\ x^2+4y^2\le 9$

$$\begin{cases} x=r\cos\varphi \\ y=\frac{1}{2}r\sin\varphi\end{cases} \Rightarrow \frac{d(x,y)}{d(r,\varphi)} = \frac{1}{2}r \qquad D\Leftrightarrow D':\ r\le 3$$

$$\iint_D (x+1)(y-2)\,dx\,dy = \int_0^{2\pi} d\varphi \int_0^3 (r\cos\varphi+1)\left(\frac{1}{2}r\sin\varphi-2\right)\frac{1}{2}r\,dr =$$

$$= -\int_0^{2\pi} d\varphi \int_0^3 r\,dr + \int_0^3 \frac{1}{2}r\,dr \underbrace{\int_0^{2\pi}\left(\frac{1}{2}r^2\cos\varphi\sin\varphi - 2r\cos\varphi + \frac{1}{2}r\sin\varphi\right)d\varphi}_{=0} = -9\pi.$$

**7.**




$$\int_{\gamma_1+\gamma_2-\gamma} P\,dx + Q\,dy = \iint_D (Q'_x - P'_y)\,dx\,dy$$

$$\int_{\gamma_1} \underbrace{y^3}_{=0}dx + x^3\underbrace{dy}_{=0} = 0 = \int_{\gamma_2} y^3\underbrace{dx}_{=0} + \underbrace{x^3}_{=0}dy$$

så $\displaystyle\int_\gamma y^3dx + x^3dy = \iint_D (3y^2-3x^2)\,dx\,dy = \{\text{polära koord.}\} =$

$$= -3\int_{-\frac{\pi}{2}}^0 d\varphi \int_0^2 r^2(\cos^2\varphi - \sin^2\varphi)\,r\,dr = -3\int_{-\frac{\pi}{2}}^0 \cos 2\varphi\,d\varphi \int_0^2 r^3\,dr =$$

$$= -3\left[\frac{1}{2}\sin 2\varphi\right]_{-\frac{\pi}{2}}^0 \left[\frac{1}{4}r^4\right]_0^2 = -3\cdot 0\cdot 4 = 0.$$

Enklare denna gång: $\gamma: \begin{cases} x=2\cos t \\ y=2\sin t\end{cases}\quad 0 \xrightarrow{t} -\frac{\pi}{2} \quad \begin{cases} x'=-2\sin t \\ y'=2\cos t\end{cases}$

$$\int_\gamma y^3dx + x^3dy = 16\int_0^{-\frac{\pi}{2}}(-\sin^4 t + \cos^4 t)\,dt = 16\int_0^{-\frac{\pi}{2}} \underbrace{(\cos^2 t+\sin^2 t)}_{=1}(\cos^2 t - \sin^2 t)\,dt =$$

$$= 16\int_0^{-\frac{\pi}{2}} \cos 2t\,dt = 8\left[\sin 2t\right]_0^{-\frac{\pi}{2}} = 0.$$