

LMA320 Flervariabelanalys

Förslag till svar på gamla tentor (MAL 600)

10 jan 05 1.  $(x, y, z) = (1, 1, 1) + t(2, -1, -1)$

2. a)  $z(x, y) = y(y^2 + 2xy)$  b)  $z = (y^2 + 2xy)^2$

3. lok. min. = 0 i  $(0, y) \forall y$ ; lok. max. =  $e^{-1}$  i  $(\pm 1, 0)$ .

4.  $f_{\max} = 4 = f(1, \frac{3}{2})$ ,  $f_{\min} = -4 = f(-1, -\frac{3}{2})$ .

5.  $\frac{7}{4} - 2 \ln 2$  6.  $\ln 2$  7.  $-\frac{1}{6} + e - \frac{1}{2}e^2$

16 aug 04 1. -3 2.  $r z'$  3. lok. min. =  $-\frac{1}{8}$  i  $\pm(\frac{1}{2}, \frac{1}{2})$ ,

lok. max. =  $\frac{1}{8}$  i  $\pm(\frac{1}{2}, -\frac{1}{2})$  (saddelpunkter  $(0, 0), (\pm 1, 0), (0, \pm 1)$ )

4.  $f_{\max} = 90 = f(0, -1)$ ,  $f_{\min} = -54 = f(-\frac{3}{2}, 0)$  5.  $\frac{\pi}{8}$

6.  $42 \sin 5 + 14 \sin 9 + 2 \cos 5 - 2 \cos 9$  7.  $\frac{\pi}{4}$

8.  $\vec{v} = (\pm 1, 0), (0, \pm 1)$  el.  $\pm(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ .  $f$  ej diff. bar i  $(0, 0)$ .

4 juni 04 1.  $(x, y, z) = (1, -1, 2) + t(26, 44, 9)$ .

3. lok. min. = -2 i  $(1, 0)$ , lok. max. = 2 i  $(-1, 0)$   
(saddelpunkter  $\pm(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ )

4.  $f_{\min} = 0 = f(1, 0, 0)$  (t.ex.)  $f_{\max} = \frac{1}{2} = f(\frac{1}{\sqrt[3]{3}}, \frac{1}{\sqrt[3]{3}}, \frac{1}{\sqrt[3]{2}})$

5.  $\frac{2\pi}{3} - \frac{2 \ln 2}{\sqrt{3}}$  6.  $2(e^2 - e)$  7.  $-\frac{7}{12}$

8. a) Ja b) Nej

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17 jan 06 1.  $a=2, b=3, c=0$  2. a)  $2\sqrt{z}' = z$   
b)  $z = \sqrt{xy} f\left(\frac{x}{y}\right)$ ,  $f \in C^1$  3.  $\lim_{x^2+y^2 \rightarrow \infty} f(x,y) = 0$ ;  
 $f_{\max} = \frac{1}{\sqrt{2}} = f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ,  $f_{\min} = -\frac{1}{\sqrt{2}} = f\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$   
4. Största värde saknas,  $f_{\min} = 2\sqrt{2} = f(\sqrt{2}, 2)$   
5.  $\frac{\pi}{6}$  6.  $\frac{\pi}{2}(1 - \ln 2)$  7.  $-\frac{8}{5}$

18 aug 05 1.  $9x - 12y + 4z = 0$  2.  $z = \frac{1}{2}(x^2 - y^2) + f(xy)$   
b)  $z = \frac{1}{2}(1 + x^2 - y^2 - x^2y^2)$  3.  $\lim_{x^2+y^2 \rightarrow \infty} f(x,y) = 0$ ;  
 $f_{\max} = \frac{1}{e} = f(\pm 1, 0)$ ,  $f_{\min} = -\frac{3}{e} = f(0, \pm 1)$   
4.  $f_{\max} = \frac{4}{\sqrt{14}} = \frac{2\sqrt{14}}{7}$  för  $(x^2, y^2) = \frac{(3, 1)}{\sqrt{14}}$ ,  $f_{\min} = \frac{1}{\sqrt{2}} = f(0, \pm \frac{1}{\sqrt{2}})$   
5.  $\frac{1}{2}$  6.  $-9\pi$  7.  $0$

27 maj 05 1.  $2x - 2y + 3z = 9$  resp.  $20x + 4y - z = 35$   
2. a)  $z = x^2 + f(xy)$  b)  $z = x^2 + xy - x^2y^2$   
3.  $\lim_{x^2+y^2 \rightarrow \infty} f(x,y) = 0$ ;  $f_{\max} = e^{-\frac{1}{2}} = f\left(\frac{1}{2}, \frac{1}{2}\right)$ ,  $f_{\min} = -e^{-\frac{1}{2}} = f\left(-\frac{1}{2}, -\frac{1}{2}\right)$   
(dessa är de enda stationära punkterna)  
4.  $f_{\max} = \frac{1}{2} = f(1, 0)$ ,  $f_{\min} = -\frac{1}{2} = f(-1, 0)$  5.  $\frac{175}{48}$   
6.  $\frac{\pi}{4}(4 - \ln 5)$  7. a)  $\vec{F} = \nabla\left(-\frac{1}{2}\left(\frac{1}{x^2} + e^{-x^2-y^2} + \frac{1}{y^2+1}\right)\right)$  b)  $0$