INTEGRATIONSTEORI (5p)
(GU$[MAF440]$ **,CTH**$[TMV100]$**)**
INLÄMNINGSUPPGIFT 1
Sista inlämningstid: Fredag 21 november kl 15.

1. (1 dp=0.1 p) Suppose $(X, \mathcal{P}(X), \mu)$ is a finite positive measure space such that X is finite and $\mu(\{x\}) > 0$ for every $x \in X$. Set

$$d(A,B) = \mu(A \Delta B), \ \ A, B \in \mathcal{P}(X).$$

Prove that $(\mathcal{P}(X), d)$ is a metric space ($[F]$, p 13), that is

$$d : \mathcal{P}(X) \times \mathcal{P}(X) \to [0, \infty[\,,$$

$$d(A,B) = 0 \ \Leftrightarrow A = B,$$
$$d(A,B) = d(B,A)$$

and

$$d(A,B) \leq d(A,C) + d(C,B).$$

2. (2 dp) $[LN]$, Exercise 4, p 10.

3. (2 dp) Let $(X, \mathcal{M}, \mu)$ be a finite positive measure space. Prove that

$$\mu(\cup_{i=1}^{n} A_i) \geq \Sigma_{i=1}^{n} \mu(A_i) - \Sigma_{1 \leq i < j \leq n} \mu(A_i \cap A_j)$$

for all $A_1, ..., A_n \in \mathcal{M}$ and integers $n \geq 2$.