INTEGRATIONSTEORI (5p)
(GU$[MAF440]$**,CTH**$[TMV100]$**)**
INLÄMNINGSUPPGIFT 2
Sista inlämningstid: Fredag 5 december kl 15.

1. (1dp) Suppose $E \subseteq \mathbf{R}$ and $E \notin \mathcal{R}^-$. Show there is an $\varepsilon > 0$ such that

$$m(B \setminus A) \geq \varepsilon$$

if $A \subseteq E \subseteq B$ and $A, B \in \mathcal{R}^-$.

2. (2dp) Let $(X, \mathcal{M}, \mu)$ be a positive measure space and $f : X \to [0, \infty]$ an $(\mathcal{M}, \mathcal{R}_{0,\infty})$-measurable function such that

$$f(X) \subseteq \mathbf{N}$$

and

$$\int_X f d\mu < \infty.$$

For every $t \geq 0$, set

$$F(t) = \mu(f > t) \text{ and } G(t) = \mu(f \geq t).$$

Prove that

$$\int_X f d\mu = \Sigma_{n=0}^{\infty} F(n) = \Sigma_{n=1}^{\infty} G(n).$$

(Hint: Write $f = \Sigma_{n=0}^{\infty} n\chi_{\{f=n\}}$.)

3. (1dp+1dp) Compute the following limits and justify the calculations:
   a)

$$\lim_{n\to\infty} \int_0^{\infty} \frac{\sin(e^x)}{1 + nx^2} dx.$$

   b)

$$\lim_{n\to\infty} \int_0^n (1 + \frac{x}{n})^{-n} \cos x dx.$$