INTEGRATIONSTEORI (5p)
(GU$[MAF440]$ **,CTH**$[TMV100]$**)**
INLÄMNINGSUPPGIFT 3
Sista inlämningstid: torsdag 29 januari 2004, kl 11.45

1. (1dp) Let $(X, \mathcal{M}, \mu)$ be a positive measure space and $f_n : X \to [0, \infty[$, $n \in \mathbf{N}_+$, a sequence of measurable functions which converges to f in measure. Prove that

$$\int_X f d\mu \leq \liminf_{n\to\infty} \int_X f_n d\mu.$$

2. (2dp) For $t > 0$ and $x \in \mathbf{R}$ let

$$g(t,x) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}$$

and

$$h(t,x) = \frac{\partial g}{\partial t}.$$

Given $a > 0$, prove that

$$\int_{-\infty}^{\infty} (\int_a^{\infty} h(t,x)dt)dx = -1$$

and

$$\int_a^{\infty} (\int_{-\infty}^{\infty} h(t,x)dx)dt = 0$$

and conclude that

$$\int_{[a,\infty[\times\mathbf{R}} \mid h(t,x) \mid dtdx = \infty.$$

(Hint: First prove that

$$\int_{-\infty}^{\infty} g(t,x)dx = 1$$

and

$$\frac{\partial g}{\partial t} = \frac{1}{2}\frac{\partial^2 g}{\partial x^2}.)$$

3. (2dp) Suppose $A \in \mathcal{R}^-$ and $f \in L^1(m)$. Set

$$g(x) = \int_{\mathbf{R}} \frac{d(y, A) f(y)}{\mid x - y \mid^2} dy, x \in \mathbf{R}.$$

Prove that

$$\int_A \mid g(x) \mid dx < \infty.$$

(Hint: $d(y, A) = d(y, \bar{A})$)