INTEGRATIONSTEORI (5p)
(INTEGRATION THEORY)
(GU[$MAF440$] ,CTH[$TMV100$])

ASSIGNMENT 2
(Must be handed in before Friday at 9 am, week 49)
Note that 1 dp=0.1p.

1. (1.5dp) Let $(X, \mathcal{M}, \mu)$ be a finite positive measure space. Prove that

$$\mu(\cup_{i=1}^{n} A_i) \geq \Sigma_{i=1}^{n} \mu(A_i) - \Sigma_{1 \leq i < j \leq n} \mu(A_i \cap A_j)$$

for all $A_1, ..., A_n \in \mathcal{M}$ and integers $n \geq 2$.

2. (1.5dp) Suppose

$$f_n(x) = n \mid x \mid e^{-\frac{nx^2}{2}},\ x \in \mathbf{R},\ n \in \mathbf{N}_+.$$

Show that there is no $g \in \mathcal{L}^1(m)$ such that $f_n \leq g$ for all $n \in \mathbf{N}_+$.

3. (0.5dp+1.5p) a) Let $(X, \mathcal{M}, \mu)$ be a complete positive measure space and suppose $A, B \in \mathcal{M}$, where $B \setminus A$ is a μ-null set. Prove that $E \in \mathcal{M}$ if $A \subseteq E \subseteq B$. b) Suppose $E \subseteq \mathbf{R}$ and $E \notin \mathcal{R}^-$. Show there is an $\varepsilon > 0$ such that

$$m(B \setminus A) \geq \varepsilon$$

for all $A, B \in \mathcal{R}^-$ such that $A \subseteq E \subseteq B$.