PREFACE

These are lecture notes on integration theory for a ten-week course at the Chalmers University of Technology and the Göteborg University. The parts defining the course essentially lead to the same results as the first three chapters in the Folland book $[F]$, which is used as a text book on the course. The proofs in the lecture notes sometimes differ from those given in $[F]$. Here is a brief description of the differences to simplify for the reader.

In Chapter 1 we introduce so called π- and λ-systems, which are substitutes for monotone classes of sets $[F]$. Besides we prefer to emphasize metric outer measures instead of so called premeasures. Throughout the course, a variety of important measures are obtained as image measures of the linear measure on the real line. In Section 1.6 positive measures in $\mathbf{R}$ induced by increasing right continuous mappings are constructed in this way.

Chapter 2 deals with integration and is very similar to $[F]$ and most other texts.

Chapter 3 starts with some standard facts about metric spaces and relates the concepts to measure theory. For example Ulam's Theorem is included. The existence of product measures is based on properties of π- and λ-systems.

Chapter 4 deals with different modes of convergence and is mostly close to $[F]$. Here we include a section about orthogonality since many students have seen parts of this theory before.

The Lebesgue Decomposition Theorem and Radon-Nikodym Theorem in Chapter 5 are proved using the von Neumann beautiful L^2-proof.

To illustrate the power of abstract integration these notes contain several sections, which do not belong to the course but may help the student to a better understanding of measure theory. The corresponding parts are set between the symbols

$$\downarrow\downarrow\downarrow$$

and

$$\uparrow\uparrow\uparrow$$

respectively.

Finally I would like to express my deep gratitude to the students in my classes for suggesting a variety of improvements and a special thank

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