# **An Historical Survey of Number Systems**

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### **1. Introduction**

In a narrow, yet highly unspecific, sense, a *number system* is a way in which humans represent numbers. We have limited our discussion already, for it is merely humans among all known species who have the ability to count and form numbers which we later can perform calculations upon. Many—often very different—number systems have been employed by many—again, very different—cultures and civilizations throughout the ages, and there still exists a wide variety of them even today, in our comparatively global society.

In a much more broad sense, a *number system* is a set of the many ways humans reason about numbers and this is the definition we will use for our discussion in this paper. But what do we mean by this above definition? Well, let us discuss the ways in which we, as humans, reason about numbers. We reason about numbers by *talking* about them, so we need a way to represent numbers in speech. We reason about numbers by *writing* about them, so we need a way to represent numbers in writing/text; this representation is known as a *notation*. Furthermore, when reasoning about numbers, we need some sort of *number base*, or *radix*, which is the fundamental number in which all other numbers stand in relation to. In recent times—we're talking perhaps a couple of centuries back in time—we also reason a lot about different *sets* of numbers that we have been able to create; therefore, the term *number system* has also been made to mean one of these sets. We will now give a gentle introduction to these four parts and put them in relation to the rest of this paper.

#### 1.1. Numbers and Speech - Number Words

*Number words* are a way for us to discuss numbers in speech. Any language generally has number words for a finite set of numbers, e.g., the numbers smaller than the number base of the number system and perhaps certain multiples of the number base, and rules to form larger ones from these. This second property of a language is not always true. Some—albeit historical and now dead—languages actually can not form numbers larger than some certain limit; most often due to lack of necessity.

This paper will describe the number words of a number system only if they are of some larger importance or of curious interest, and even then in a very abstract manner, not indulging ourselves to make a linguistic survey.

#### **1.2.** Numbers and Writing — Number Notation

*Number notation* is the way in which we form numbers in text. The notation is often tightly linked with the language being employed as there is often a connection between the way words are formed in speech and in the way they are formed in writing. This is, however, not always the case. And even if it is, there may exist many notations for use depending on context and application. The notation may also change greatly over time, even if the number system as a whole changes little—often depending on other achievements in society, such as writing material.

Notation is of central importance in this paper. Most of our discussion will revolve around the notation used by different societies through history. There are three reasons for this. First, and somewhat trivial, it is the cornerstone in our understanding of number systems from past times, as it is more or less the only record we have of them. Second, notation tells a lot more about the mathematical advancement of the civilization in which it was used than do number words—if their use has been preserved. Third, notation is such a beautiful thing. Your author truly loves the written word. Not so much the content,

rather the actual building blocks that forms it.

#### **1.3.** Numbers in Relation — Number Base

A *number base* is a number in which other numbers stand in relation to. Numbers are expressed in a certain number base and calculations are performed using the values of a number given its base. Throughout the ages there have been an exceedingly large number of *radices*—a somewhat more scientific term with the same meaning as number base—by which mathematicians have performed their calculations. Perhaps surprisingly, and we are jumping the conclusions somewhat, our decimal—base 10—number base, where all numbers are expressed as multiples of ten, is not the oldest, nor the one most often used.

The number base is of grave importance to the possibility of further advancement of a number system and the mathematics utilizing it. As we will see, a number base can stagnate a whole civilizations mathematical progression, and we will therefore give it an important role in our discussion.

#### **1.4.** Sets of Numbers — Number Sets

*Number sets* are sets of numbers that we are able to perform mathematical operations upon. As we will not discuss them further, we give some examples of them right here and rid ourselves of them: natural numbers ( $\mathbb{N}$ ), real numbers ( $\mathbb{R}$ ), rational numbers ( $\mathbb{Q}$ ), algebraic numbers, complex numbers (1 + i), p-adic numbers (1.1), surreal numbers, hyperreal numbers, quaternions ( $\mathbb{H}$ ), and so on.

Number sets play no important role in our discussion and we will hopefully not have to discuss them any further.

### **1.5.** Outline of this paper

In the sections following this one, we will give a chronological survey of number systems throughout our mathematical history. We begin with prehistoric number systems, which—in a way—predate even counting. We then travel through the proverbial wormhole, if you will, and arrive at about ~3000 B.C.E. where the Egyptians are ruling supreme. Not alone though, as the Summerians are more or less side-by-side and show signs of a much higher mathematical understanding (~3000 B.C.E.). As the Summerians more or less become the Babylonians around 2000 B.C.E., we discuss them next. The Chinese, working largely in isolation, are next: ~2500 B.C.E. Greek mathematically impaired Romans provide no real mathematical interest, but notationally they are of large interest (~0 C.E.). Four hundred years later (~400 C.E.) and across the globe, the astrologists of the Mayan civilization are making incredible computations using, what seems to be, a number system developed in isolation. The Hindus make achievements during a long period of time. Perhaps the most important ones to us around 300 B.C.E. to 400 C.E. Then again, we make a rather big jump in history and arrive in modern times, where we discuss computers and number systems. We conclude by drawing some conclusions of our discussion.

# 2. Primitive and Prehistoric Number Systems

Primitive and Prehistoric number systems are number systems that have been developed when there has been some need to express magnitudes. Examples of these are what is known as *body-count*, the "bundle-of-sticks" method, tally sticks, 2-count, and neo-2-count. Although exceedingly primitive and limited in applicability, these systems have been the beginning of counting in almost every civilization, and many are still used by many of the primitive—in the sense of development, not ability to reason—cultures on Earth today. We will make a quick survey of these number systems and discuss their uses and their limitations.

#### 2.1. The "bundle-of-sticks" Method

This method—for it is hardly a number system in its own right—is a way to represent magnitudes. It works by mapping a set of unmanageable items, say coconuts, unto something more manageable, say sticks. So, for each coconut we select a stick. When we have gone through all our coconuts, we will have the total number of coconuts represented by a bundle of sticks. This may not be counting in any real sense of the word, but at least we have arrived at a magnitude that is easier to deal with.

The "bundle-of-sticks" method can not be used to express anything but quantities, and can perhaps not even be considered a way to count. Nevertheless, it has been a method often employed for applications like the one described in the previous paragraph.

#### 2.2. 2-Count

2-count is what could perhaps be called a *counting system*, as it provides a way to count, yet provides no means to perform computations. It works as follows: There are number words for numbers 1 and 2, and sometimes an additional for expressing "many", i.e., an uncountable magnitude. These are then combined to express larger quantaties. This is, however, limited to small multiples of 2, as beyond a certain limit, counting the number of times that the word for 2 has been repeated would require a more advanced system in itself! This method of counting is seen across the globe even today, among primitive cultures, and many conjecture that it is so old that it has been with us since the first human civilizations began arising before people spread out on Mother Earth. Table 1, "Bushman 2-Count" shows the number words used among the Bushmen in South Africa.

Number	Word or Combination
1	ха
2	t'oa
3	'quo
4	t'oa-t'oa
5	t'oa-t'oa-ta
6	t'oa-t'oa-t-oa

#### Table 1. Bushman 2-Count

As already stated, two-count is rather limited in applicability. Beyond a certain limit, it can not be used anymore, as the number of repetitions become too large to keep track of. They have, however, played an important role in developing more usable systems and is the ancestor of the neo-2-count system discussed in the next section.

#### 2.3. Neo-2-Count

Neo-2-count is an extension to 2-count, where words for 3 and 4 have been added, and larger multiples of 2 are expressed as multiplications of 2 with 3 or 4. The Toba in Paraguay use the system outlined in

Table 2, "Toba Neo-2-Count".

Number	Word or Combination
1	nathedac
2	cacayni <i>or</i> nivoca
3	cacaynilia
4	nalotapegat
5 = 2 + 3	nivoca cacaynilia
$6 = 2 \cdot 3$	cacayni cacaynilia
$7 = 1 + 2 \cdot 3$	nathedac cacayni cacaynilia
$8 = 2 \cdot 4$	nivoca nalotapegat
$9 = 2 \cdot 4 + 1$	nivoca nalotappegat nathedac
$10 = 2 + 2 \cdot 4$	cacayni nivoca nalotapegat

 Table 2. Toba Neo-2-Count

There are several variations to this scheme, but all amount to a simplification of expressing larger numbers. Of most interest, perhaps, is a system where groups of 4 is used, instead of 2. Still, these systems all share the same limitations as basic 2-count, in that they have an inherit limit in expressiveness.

#### 2.4. Body Counting

Body Counting is another method for describing magnitudes. Body parts are associated with a given quantity and by touching them, the given quantity is inferred. It can be as limited to the ten fingers or as advanced as upwards of forty different parts of the body. Again, this system is constrained to a fixed limit, yet is an advancement over both types of 2-

Again, this system is constrained to a fixed limit, yet is an advancement over both types of 2count. Body counting only occurs in areas prone to 2-count.

#### 2.5. Tally Sticks

Not only limited to notches on sticks, *tallies* are most likely the oldest means of recording quantaties in human history. It is also the one that has stuck with us the longest, even being used for accounting well into the 1800th century. A tally stick is simply a piece of wood in which notches have been made, representing some quantity. More advanced forms have different notches, e.g., deeper grooves or crossed notches, to represent larger quantities. For recording some transfer of goods, talies where sometimes made and split in two along the tally so that later the transfer could be validated by both parties. When used for recording debts, tally sticks where often made shorter as parts of the debt were payed, removing notches corresponding to the returned amount.

Not much more than a way to record transactions and quantities, tally sticks nonetheless have played an important role in many civilizations, as the need for such records have become apparent. Many are truly a beautiful piece of history, as their importance often warranted great care in their creation.

# 3. The Egyptian Number System

The first signs of the Egyptian *hieroglyphic* number system date to around 3000 B.C.E. It uses 10 as a radix, thus it is a decimal system. Number words are not known, but numerals have been preserved, and our knowledge of their use for performing calculations has been in large thanks to two surviving papyri, namely the *Rhind* and the *Moscow*. Actually, that's not quite true, as they are both written using a more "informal" system, known as the *hieratic form*. There is even a third form, used by scribes in later eras, known as the demotic form.

The hieroglyphic form, shown in Table 3, "Egyptian Hieroglyphic Numerals" uses symbols for powers of ten, relying on repetition for other numbers. As the Egyptians wrote from right to left, the largest power of ten appears to the right of the other numerals. There is, however, no possibility for misinterpretation, as it is a purely additive system in which the number is read out by adding the numerals constituting it.



Table 3. Egyptian Hieroglyphic Numerals

The hieratic and demotic form were somewhat simpler and had more symbols. They had unique symbols for 1 through 9, 10-90, 100-900, and 1000-9000. This system, where numerals have a unique cipher, is more advanced than the simple additive one, and makes for easier calculations. The Egyptians have not shown any signs of utilizing this added benefit, though, and its importance in the development of more advanced systems has been largely neglected [Boyer]. As numbers 2 and 3 are repetitions of 1, the system was not fully ciphered either. Figure 1, "Egyptian Hieratic Numerals" shows the hieratic numerals.

1	10 1	100 ,-•	1000 5
2	20 3	200 🔑	2000 🦊
3 <b>11</b>	30 <b>Š</b>	300 🔎	3000 🗳
4	40 🗲	400 뀓	4000 🎢
5 7	50 1	<b>تن</b> ر <sup>500</sup>	5000 <b>ž P</b>
° <b>L</b>	60 堂	600 🎢	6000 🖈
7 4	70 ┨	700 7	7000 🖌
8 📲	80 🚔	800 🙇	8000 🚜
9 <b>A</b>	90	900 ,3	9000 🏂
Hieratic numerals			

**Figure 1. Egyptian Hieratic Numerals** 

In summary, performing calculations using the hieroglyphic form was tedious at best: multiplications had to be perform iteratively, and the Egyptians only used unit fractions, with the exceptions of 2/3 and 3/4. Expressing a fraction was done by adding an additional hieroglyph to the numeral, which is simple enough, but also restrictive enough to make fractional calculations exceedingly hard to perform in practice.

### 4. The Sumerian and Babylonian Number System

Ah, the Sumerians with their base 60 and their place-value system, what marvels of human enginuity. The Summerians lived in the regions of the lower Tigris and Euphrates valleys around 3000 B.C.E. Later on, as the importance of the city of Babylon grew, power was shifted upstream and the Sumerians more or less became the Babylonians. Such is history, and we will therefore deal with them together—furthermore, their number system is more or less the same, differing only in their symbols of notation and some small additions to aid in more advanced calculations.

The Sumerians used 60 as its number base, known as a sexagesimal system. Numerals were written from left to right and they used unique symbols for numbers 1, 10, and 60. They also had symbols for a couple larger multiples of 60, but we will not bother discussing them. The Sumerian numerals are shown in Table 4, "Sumerian Numerals". The symbol for 60, which is derived from that of 1, is slightly larger.



**Table 4. Sumerian Numerals** 

The Babylonians used a slightly different form, known as *cuneiform*. It uses only two symbols, 1 and 60 share symbols—not even its size is made unique. Initially, this may seem to introduce an enormous amount of ambiguity, but as we'll show, this isn't really a problem. First, however, we show the cuneiform in Table 5, "Babylonian Cuneiform Numerals".



**Table 5. Babylonian Cuneiform Numerals** 

As the entry for 600 in Table 5, "Babylonian Cuneiform Numerals" displays, the value of the wedge representing both numbers 1 and 60 depends on its relation to other symbols in the numeral. This is what is known as a *place-value number system*. It is perhaps the biggest advance into a higher order of mathematics in number system history. There is however some ambiguity here, as found equally mean 70 (60 + 10). Furthermore, as fractions and large powers of 60 came into general use, found equally mean 70 multiplied or divided by any power of 60; its value being inferred based on context.

The main problem with their system is that it is place-value but without a zero, so we have

additional ambiguity in that we can not distinguish 64 from 3604. An additional symbol was later introduced, indicating an empty space between two digits in a numeral. It looked like two symbols for 10, one transposed upon the other. This symbol was not used at the end of numbers, though, so there was still the ambiguity of a number being multiplied by any power of 60. Even later, a real zero was introduced, represented by a small circle, that could appear at the end of numbers as well.

Why the Sumerians/Babylonians chose 60 as their number base is still unknown. Perhaps the most likely explanation is its divisibility by 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30. It is by far the largest base ever used in a number system of historical relevance.

### 5. The Chinese Number System

The chinese number system dates back to around 2500 B.C.E. and has changed little over the years, the only real important change being the addition of the zero—which was not as important as it may initially seem. Many scholars believe that it was developed largely in isolation and that only much later were influences of outer achievements made apparent in the Chinese number system.

Chinese has number words for 1 through 9 and for various powers of ten, much like English. Their way of forming larger numbers is also much the same as it is done in English, where each number is attached to a given multiple of ten, e.g., hundred, thousand, and so on. The system is, however, not a positional system, so words can be spoken in any order without changing the meaning of the number as a whole.

The number notation is much like the number words. There are separate ideograms for 1 through 9 and for various multiples of ten. Thus, this is not a positional system. In fact, there is not one unique numeral, but a set of them, each with different applicability and some also of historical significance. Common, however, is that numbers are written in the same way as they are spoken, e.g., 78000 would be expressed as  $\pm \pi \pi + \pi$ , or "seven ten-thousands plus eight thousands". Of course, this isn't how it would look in a Chinese book on mathematics, for text is written from top to bottom, left to right on a page, which will not be reproduced here. Table 6, "Chinese Numerals" lists two of the five forms available. It is important to note that the ideograms constituting the formal writing form are complex for a good reason: they were originally intended for use on banking notes to avoid falsification. Not shown here are the counting-rod form used to represent numbers on a counting board. It is a truly beautiful form that should have received closer treatment in this paper.

Number	<b>Casual Writing Form</b>	Formal Writing Form
0	$\bigcirc$	零
1	<b>—</b>	志 豆
2		叁
3	<u> </u>	肆
4	四	肆
5	Ŧī.	伍
6	六	陸
7	七	柒
8	八	捌
9	九	玖
10	+	拾
100	百	佰
1,000	千	仟
10,000	万	萬

From the previous paragraph, it should be obvious that the Chinese did not need a zero, as numbers are explicitly given their "position". This is what some refer to as a *named position-value system*. It is also fully ciphered, which, as previously stated, helps out a lot when performing calculations.

### 6. The Greek Number System

The Greek number system of around 500 B.C.E. is known as the Ionian number system, was mostly likely inspired by the demotic number system of the Egyptians. It is a decimal number system and comes with two notations, one older, cumbersome and limiting, and a newer one using twenty-seven symbols of the alphabet—which was actually only comprised of twenty-four symbols, so three were borrowed from an older alphabet. We will only discuss the newer form, as the old one is much like that of the Romans, albeit even the old one is more advanced than the Roman one—and they ruled Europe for how long?, which is discussed in Section 7, "The Roman Number System".

Table 7, "Greek Numerals" show the Ionian numerals. Symbols for 6, 90, and 900 are the older letters we discussed in the previous paragraph. Now, after looking at them, you might reason that it must have been cumbersome to remember twenty-seven unique symbols and also separate them from surrounding text as numbers. Well, guess what, it does not seem to have been a big issue with the Greeks. They did provide two means of separating numbers from surrounding text by either priming the sequence of letters, or overlining them. Furthermore, the twenty-seven unique symbols of the thus fully ciphered system aren't much harder to remember than the ten we are forced to reckon with.

•	1	10	100	1000
1	α	ι	ρ	,α
2	β	к	σ	,β
3	γ	λ	τ	,γ
4	δ	μ	υ	,δ
5	3	ν	φ	,8
6	ς	ц	χ	,ς
7	ζ	0	ψ	,ζ
8	η	π	ω	,η
9	θ	4	シ	,θ

**Table 7. Greek Numerals** 

The two great improvements here are that it is a fully ciphered system and that one, and only one, radix is being used, which is a great improvement over the Babylonians, who had a sort of middle base at 10. Here, as well (see Section 5, "The Chinese Number System"), the need for zero has been alleviated by the use of a named position-value system.

### 7. The Roman Number System

Not much of algebraic interest can be said about the Romans, other than that they were great in numbers and controlled a vast empire for hundreds of years, which at least counts for something. They did however have a beautiful way of writing numbers. Primite, yet highly artistic in some sense, they, as the Greeks (see Section 6, "The Greek Number System"), used letters of the alphabet to form numbers. In fact, they did not even employ a method of separating numbers from surrounding text—simply stunning. Yet, when you look at Roman inscriptions, it just seems so right.

Anyway, the Roman number system came into use at the time around the beginning of the common era, i.e., ~0 C.E. It was also decimal and was not a place-value system, rather an additive like the Egyptian one (see Section 3, "The Egyptian Number System"). They did, however, have one improvement over the Egyptians—at least for the additive part of writing numbers; for performing calculations this was a horrible mistake—they had numerals for half-values of ten. This reduced repetition, but for performing computations, this is retarding in nature. As a side note, the Romans did not use the system of subtracting quantities just below the larger one for writing numerals, e.g., IV for 4. This was a later invention, long after the fall of the Roman empire. Furthermore, a common misconception is that the Romans did not have numerals beyond one thousand. They did, they actually had a special symbol for multiples of 100,000, though it only came into use in a later stage of the civilization. Table 8, "Roman Numerals" show the Roman numerals and includes the special symbols just discussed.

Number	Symbol
1	Ι
5	V
10	Х
50	L
100	С
500	D
1,000	M, $(D, CID, or \infty)$
5,000	D or ICC
10,000	(D) or CCIDD
50,000	CCCI
100,000	CCCIDDD
500,000	V
1,000,000	X

**Table 8. Roman Numerals** 

The origin of D and M for 500 and 1000 are actually hinted at in the older symbol for 1000, namely take the right half of it and you get a D, or 500. The origin of X is also suggested in the common practice of crossing two I's on tally sticks to denote 10. V's form is suggested as the upper half of an X and it seems that only C and M have some connection to the language, i.e., *centum* and *mille*.

As suggested above, the Roman number system is extremely primitive for its time and one may wonder why this is so. Presumably, mathematics was uninteresting to the Romans, beyond simple calculations and quantities.

# 8. The Mayan Number System

Around 400 C.E. the Maya Indians develop their number system in conjunction with growing interest in the calendar. The number system was based on the number 20, known as a *vigesimal system*, with 5 as an intermediate base. The system included three symbols for numbers 1, 5, and 0. That's right. They did not have a symbol for their base, but they did have one for zero. It seems as if this was their own invention. Their system was also a place-value system, thanks to their zero. Figure 2, "Mayan Numbers" shows a set of mayan numbers, using these numerals. This is not how they would appear on stone, as their inscriptions were much more ornamental, but gives an idea of how they formed numbers.

0	1	2	3	4
$\bigcirc$	•	••	•••	••••
5	6	7	8	9
_	•	••	<u>•••</u>	<u></u>
10	11	12	13	14
_	÷		<u>•••</u>	<u></u>
-	_	_	-	-
15	16	17	18	19
	•	••	•••	••••
$\equiv$	$\equiv$	$\equiv$	$\equiv$	$\equiv$
20	21	22	23	24
•	•	•	•	•
$\bigcirc$	•	••	•••	••••
25	26	27	28	29
•	•	•	•	•
_	<u>.</u>		•••	

Figure 2. Mayan Numbers

Their number system was almost exclusively used for calculations pertaining to their calendar, where it served them well. Their calculations of the length of a year was more exact than our modern one (the Gregorian calendar).

# 9. The Hindu Number System

The Hindu number system as we know it today—for it is more or less the one we (as in us Westerners) use today as well—can be attributed to two men: the great hindu astronomer Âryabhata and his pupil Bhâskara I, during the period 499-522 B.C.E. Âryabhata invented a system that was based on using syllables and combining them to form greater numbers. Bhâskara I simplified the system somewhat and made it decimal place-value system. The Hindus had used a decimal system for a long time and even had the zero before this time, but it was not until these two astronomers came up with their system for forming numbers that the system was more or less complete.

As all Hindu literature was written on verse form, they also had to come up with a system to form numbers that fit well in verse. Thus, they substituted different words for numbers. Examples are "moon" for one, as there is only one moon, "eyes" for two, as we have two arms, and so on. 201 would then be *paksa-kha-eka*, or *eyes-hole-one*.

The zero was very important in the development of the Hindu number system, as it allowed for the introduction of a place-value decimal system using only nine other symbols. This made calculations much simpler than any previous system could have provided. The zero later reached the Arabs, who were not very fond of it. Even the great al-Khowârîzmî-who has given name to the word "algorithm"— considered there to be only nine numerals, even though he made use of the zero.

For completeness sake, we provide a table over Devanagari numerals in Table 9, "Devanagari Numerals". Remember, though, that Devanagari is only one of a multitude of different scripts in the Hindu region and is merely an example of Hindu numerals.

Number	Devanagari
0	0
1	8
2	२
3	३
4	8
5	ૡ
6	द
7	9
8	٢
9	S

**Table 9. Devanagari Numerals** 

# **10. Modern Number Systems**

In our modern society, almost everything uses the Hindu-Arabic number system, which we ascribe to the Hindus of more than a millenia ago. We are quite happy with it, and the number base of ten works fine and there is even a correlation between it and the number of fingers on our two hands. Yet, there are an exceeding amount of calculations performed in our society in three other bases. These calculations are performed by computers and they do not count on their circuits, or they would have to use a very large number base. They prefer calculations in base 2, as a number can then be represented easily by switching circuits on and off. Many computations can also easily be made by switching the right circuits and this makes for a great model of arithmetic calculation for a computer.

It doesn't work great for humans though. We have a hard time reading long sequences of 1's and 0's, so people who are "forced" to work with computers, known as *programmers*, prefer to deal with numbers in base 8 and 16. Base 10 is often also available, but base 8 and 16 are good to work with since they require only 3 or 4 bits of information each (000-111 and 0000-1111 respectively). This is not of great importance to most humans, but programmers have to be somewhat proficient with these bases as well, as a lot of documentation will use them and many computations performed on a computer are more easily expressed in base 8 or 16.

### **11. Future Number Systems**

Well, we can not see into the future, for if we could it would not be the future any more. Yet, we can make a guess at what the number systems of the future may look like. Although our modern decimal place-value, fully ciphered number system has worked miracles in the achievements of mathematics, for every day calculations, the base of 10 is not always the best, for it is only divisibly by 1, 2, and 5. Some argue that a base of 12 would work better, as it is divisible by 1, 2, 3, 4, and 6. which could make many calculations a lot easier to perform, at least in ones head. In other parts of our number systems, however, there seems to be a stability that implies that not much will change in those respects. It has evolved over many millenia, and perhaps it has reached some sort of perfection (which is a very dangerous thing to say, I'm sure the Romans thought they had done so as well). But not seeking for better representations would be foolish, as history have shown that great leaps in our understanding of numbers have been made based on seemingly simple ideas.

# References

[Boyer] Boyer, C.B. "Fundamental Steps in the Development of Numeration", Isis, 35, 157–158.