

MAN460: list of theorems that may appear in exam

Note that some *exercises* in the exam could be of theory like type, without being in this list below. These would be problems of the type

- *Consider the following problem prove that the solutions are positive for all times*

Note that below, unless otherwise stated, I expect that you are able to state AND prove the corresponding theorems. Note that I have not given exact references in the books if you don't find it in your book, please ask me where to look.

1. Formulate and prove a theorem on the existence and uniqueness of solutions to first order differential equations, assuming a Lipschitz condition (*note that this is a rather vague statement ... it is part of the problem to give it a mathematically exact form*)
2. prove that the Euler polygon method (the Cauchy Euler method) converges
3. Prove that the set of solutions to a linear, homogeneous differential equation form a linear space. What is the dimension?
4. Properties of the matrix exponential: definition, convergence of power series, proof of the formula $\exp(A + B) = \exp(A)\exp(B)$ if A and B commute.
5. Matrix norms. Estimates of $\exp(At)$ in terms of the eigenvalues of A .
6. Statement of the Cayley-Hamilton theorem, statement of the Jordan form theorem for matrices.
7. The Gronwall lemma
8. Construction of a fundamental matrix for an arbitrary constant coefficient linear system.
9. Definition of stability, asymptotic stability, exponential stability.
10. A stability theorem and an instability theorem for systems of the form $\mathbf{y}' = A\mathbf{y} + \mathbf{g}(\mathbf{y}, t)$.
11. Lyapunov's stability theorem.
12. Definition of Sturm's boundary value problem (and of the corresponding eigenvalue problem)
13. Existence (and definition) of a Green's function to such boundary value problems
14. Definition of scalar products, of self adjoint operators, proof that selfadjoint operators have real eigenvalues