MATEMATIK
Göteborgs universitet

Tentamensskrivning i **Matematisk logik** (MAN 480) 5p
2002-01-18, kl 8.45-13.45.
Inga hjälpmedel (engelskt-svenskt lexikon får användas).
Telefonvakt:

All answers and solutions must be carefully motivated!

1. **a)** Let φ, ψ, and σ be propositional formulas. Show that $\varphi, \psi \models \sigma$ if and only if $\models (\varphi \wedge \psi) \rightarrow \sigma$. (1.5 p)

   b) Show that $\{p_0, \neg p_1, \ldots, p_{2n}, \neg p_{2n+1}, \ldots\}$ is consistent. $p_0, p_1, \ldots$ are the propositonal symbols. (1.5 p)

2. Give a derivation in Natural Deduction of

   a) $\neg\neg p \rightarrow p$. (1p)

   b) $(p \vee \bot) \rightarrow p$. (1.5p)

   c) $\varphi(a) \rightarrow \forall x \varphi(x)$ from the assumption $\forall x(x = a)$, where a is an individual constant. (2p)

3. Let Γ be a set of sentences. Show that the following three statements are equivalent

   (i) Γ is consistent.

   (ii) For no φ, $\Gamma \vdash \varphi$ and $\Gamma \vdash \neg\varphi$.

   (iii) There is at least one φ such that $\Gamma \nvdash \varphi$. (3p)

4. Give a Kripke model in which

   a) $\neg\neg p \rightarrow p$ is false. (1p)

   b) $\neg(\neg p \vee \neg q \vee \neg r) \rightarrow (p \wedge q \wedge r)$ is false. (2p)

5. For each of the following sentences, give a structure in which the sentence is false. P, Q and R are predicate symbols.

   a) $\forall x \exists y P(x, y) \rightarrow \exists y \forall x P(x, y)$ (1.5p)

   b) $(\forall x Q(x) \rightarrow R) \rightarrow \forall x(Q(x) \rightarrow R)$ (1.5 p)

6. **a)** Let T_1 and T_2 be two theories. Show that, in general, $T_1 \cup T_2$ need not be a theory. (1.5 p)

   b) Show that if Γ has only finite models, then there is an n such that each model has at most n elements. (2p)

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7. Give a prenex form to $(\forall x Q(x) \to \exists x Q(x)) \wedge P(x, y)$, where P and Q are predicate symbols. (2p)

8. Define Gödel's formula U and show that if $\mathcal{N}$ is ω-consistent then neither U nor $\neg U$ can be derived in $\mathcal{N}$. ($\mathcal{N}$ is Peano's arithmetic.) (3p)

The exam is expected to be corrected within 10 days; when it is corrected I will put a message on the homepage of the course.

Good luck!

Jan