A few facts you need to know to be able to read the extra material on Gödel's incompleteness theorem

Last change: 10 December

- The set of natural numbers is denote by D_N.
- Peano's axiom are denoted by $\mathcal{N}$ and $\vdash_{\mathcal{N}} A$ means that the formula A is derivable from $\mathcal{N}$.
- The function symbol for the successor function is $'$, van Dalen uses S.
- A k-place relation R on the natural numbers is *representable* in $\mathcal{N}$ by the formula $A(x_1, ..., x_k)$ if the following two conditions hold:
  - if $R(n_1, ..., n_k)$ then $\vdash_{\mathcal{N}} A(0^{(n_1)}, ..., 0^{(n_k)})$,
  - if $R(n_1, ..., n_k)$ is false then $\vdash_{\mathcal{N}} \neg A(0^{(n_1)}, ..., 0^{(n_k)})$.
- A k-place relation R is *decidable* if there exists an algorithm $Decide_R$ (i. e. a program in your favourite programming language), such that
  - if $R(n_1, ..., n_k)$ then $Decide_R(n_1, ..., n_k) = T$,
  - if $R(n_1, ..., n_k)$ is false then $Decide_R(n_1, ..., n_k) = F$
- We will use the following fact without proof:

  If the relation R is decidable, then it is representable in $\mathcal{N}$.