Mathematical Sciences (MV), GU
Thursday, March 16, 2017, 14:00-18:00.

No aids (closed book, closed notes).
Phone: 7725385/0768845063
Presence of teacher: ∼15:15 and ∼16:30

Exam in MMA 100 Topology, 7.5 HEC.

1. Prove or disprove the following claims for general topological spaces: (a) The set of inner points A^o is the same as that of its closure, $A^0 = (\bar{A})^o$. (b) The inverse image $f^{-1}(C)$ of a closed subset $C \subset Y$ of a map $f : X \to Y$ is closed in X.

2. Prove that a compact subset C of a Hausdorff space X is closed.

3. Let (X, d) be a connected metric space consisting of at least two points. Prove that there exists a continuous function $f : X \to [0, 1]$ which is *onto*.

4. Recall that a wallpaper group G is a discrete subgroup of motion group $\mathbb{R}^2 \rtimes O(2)$ such that the orbit space $\mathbb{R}^2/G$ is a compact Hausdorff space. (a) Prove that if G contains a rotation R_θ of angle θ then $\theta = 2\pi r$ for some rational number r. (b) Prove that if G is a wallpaper group such that $\mathbb{R}^2 \to \mathbb{R}^2/G$ is a covering then G contains no reflections or rotations, i.e. $G \cap O(2) = \{\mathrm{Id}\}$. (Hint: Consider a neighborhood of the origin in $\mathbb{R}^2$.)

5. Let X be the set of pairs of distinct complex numbers, $X = \{(c_1, c_2) \in \mathbb{C}^2; c_1 \neq c_2\}$. Prove that X is path connected and find its fundamental group. (Hint: Consider the subset $X_0 = \{(c, 0) \in \mathbb{C}^2; c \neq 0\}$ of X.

6. Let $f : X \to S^n$ be a map which is not onto. Prove that f is homotopy to a constant map $e_p : X \to S^n, x \mapsto p$ for some $p \in S^n$.

7. Let $\mathbb{P}^n = \mathbb{R}^{n+1}/\sim$ be the projective space of lines $[u] = \mathbb{R}u$ in $\mathbb{R}^{n+1}$, $n > 1$; observe that $\mathbb{P}^1$ is homeomorphic to the unit circle. Consider the map $f : \mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^3$, $([u_1, u_2], [v_1, v_2]) \mapsto [u_1v_1, u_1v_2, u_2v_1, u_2v_2]$. Prove that f is well-defined injective map. Describe the induced group homomorphism $f_* : \pi_1(\mathbb{P}^1 \times \mathbb{P}^1) = \mathbb{Z} \times \mathbb{Z} \to \pi_1(\mathbb{P}^3) = \mathbb{Z}_2$.

8. Formulate the definition that $\pi : X \to Y$ *is a covering map*. Prove that a covering map π is an open map, i.e., π maps open sets to open sets. Is it always a closed map?

8 problems, 24 points $= 8 \times 3$. Grade limits: 12p for Godkänd (Pass), 18p for Väl Godkänd (Very Good).

GZ