MMA 100 Topology

June 10, 2019

Solutions.

1. N, N, Y, Y, Y.

2. This is a minor variant of the proof in Example 1 on page 103.

3 (a) As done in class,

let $X = \mathbb{R}^{\omega}$ in box topology,

$A = \{(x_1, x_2, x_3 \ldots) : x_i > 0 \ \forall i\}$ and

$x = (0, 0, 0, 0, \ldots)$; $x \in \bar{A}$ since every basis element containing x intersects A. But there is no sequence in A converging to x (as proved in class).

(b) No. $\forall n$, let $x_n \in B(x, \frac{1}{n}) \cap A$.

Then easy to show $x_n \to x$.

(Given a basis element $B(x, \varepsilon)$ around x, choose n s.t. $\frac{1}{n} < \varepsilon$.

Then $x_n \in B(x, \varepsilon)$.)

4a. See book

b. There is no such f since $f(X)$ would then be connected

But then $(-\infty, 23)$ and $(23, \infty)$ would be a separation of $f(X)$.

c. Disconnected. Letting $A = \{ (x_i) : \sup_i |x_i| < \infty \}$, A, A^c is a separation.

If $x \in A$, $B(x, 1/2) \subseteq A \Rightarrow A$ open.

If $x \notin A$, $B(x, 1/2) \subseteq A^c \Rightarrow A^c$ open

5. (a) In the product topology, it is compact by Tykenof's Theorem.

It is not compact in the uniform topology. One way to see this is,

since $\{0,1\}^{\mathbb{N}}$ is a closed subset, it is enough to show $\{0,1\}^{\mathbb{N}}$ is not compact.
But each element in $\{0,1\}^{\mathbb{N}}$ is open in $\{0,1\}^{\mathbb{N}} \Rightarrow$ not compact.
Since ~~it~~ the box topology is finer than the uniform top., it is also not compact

(b) $[0,1]^{\mathbb{N}}$, $\{x : \lim_{i\to\infty} x_i = 0\}$, A

(c) metrizable, metrizable, not metrizable

6. See book and lecture

7. See book.

8.

(a) $x \to \left(e^{-3\pi i x}, e^{\frac{\pi i x}{4}}\right)$ is easily verified to be a lifting of g.

(b) $\pi(S^1 \times S^1)$ is identified with $\mathbb{Z} \times \mathbb{Z}$ in the natural way. With this identification,

$f^*((1,0)) = (-2,0)$ and $f^*((0,1)) = (0,4)$

So $f^*(n,m) = (-2n, 4m)$.

The image of f^* is the subgroup $2\mathbb{Z} \times 4\mathbb{Z}$ of $\mathbb{Z} \times \mathbb{Z}$.