INTEGRATION THEORY (2010)
(GU$[MMA110]$ **,CTH**$[tmv100]$**)**

ASSIGNMENT 1
(Must be handed in at the latest Tuesday at 11^{45}, week 38)
(6 p = 1 credit point)

1. (1 p) Suppose $(X, \mathcal{P}(X), \mu)$ is a finite positive measure space such that $\mu(\{x\}) > 0$ for every $x \in X$. Set

$$d(A,B) = \mu(A \Delta B),\ \ A, B \in \mathcal{P}(X).$$

Prove that

$$d(A,B) = 0 \ \Leftrightarrow A = B$$
$$d(A,B) = d(B,A)$$

and

$$d(A,B) \leq d(A,C) + d(C,B).$$

2. (0.5 p+0.5 p) Let $\theta{:}\mathcal{P}(X) \to [0,\infty]$ be an outer measure. (a) Suppose $Y \subseteq X$ and set $\varphi(E) = \theta(E \cap Y)$ if $E \subseteq X$. Prove that $\varphi{:}\mathcal{P}(X) \to [0,\infty]$ is an outer measure. (b) Suppose $X = \mathbf{R}$ and θ is a metric outer measure. Show that φ is a metric outer measure.

3. (1 p) Suppose μ is a σ-finite positive measure on $\mathcal{R}$. Prove that the set of all $x \in \mathbf{R}$ such that $\mu(\{x\}) > 0$ is at most denumerable.