INTEGRATION THEORY (2010)
(GU$[MAF440]$**,CTH**$[TMV100]$**)**

ASSIGNMENT 2
(Must be handed in at the latest Tuesday at 11^{45}, week 40)
(6 p = 1 credit point)

1. (1 p) Compute

$$\lim_{n\to\infty}\int_0^n (1+\frac{x}{n})^n e^{-2x}\cos x dx.$$

2. (1 p) Suppose $(X,\mathcal{M},\mu)$ is a positive measure space and $(Y,\mathcal{N})$ a measurable space. Furthermore, suppose $f: X\to Y$ is $(\mathcal{M},\mathcal{N})$-measurable and let $\nu=\mu f^{-1}$, that is $\nu(B)=\mu(f^{-1}(B))$, $B\in\mathcal{N}$. Show that f is $(\mathcal{M}^-,\mathcal{N}^-)$-measurable, where $\mathcal{M}^-$ denotes the completion of $\mathcal{M}$ with respect to μ and $\mathcal{N}^-$ the completion of $\mathcal{N}$ with respect to ν.

3. (1 p) Suppose $f\in L^1(m)$, where m is Lebesgue measure on $\mathbf{R}$. Prove that the series $\Sigma_{k=-\infty}^{\infty} f(x+k)$ converges for m-almost all x.