

INTEGRATION THEORY (2010)
(GU[MAF440], CTH[TMV100])

ASSIGNMENT 3

(Must be handed in at the latest Tuesday at 11⁴⁵, week 42)
(6 p = 1 credit point)

1. (1 p) A Borel probability measure μ on \mathbf{R}^n is given by the equation $d\mu = a \exp(-|x|^n) dx$, where a is a positive constant. Find the μ -measure of the Euclidean ball $B = \{x \in \mathbf{R}^n; |x| \leq 1\}$.

2. (1 p) Let $I =]0, 1[$ and

$$h(x, y) = \frac{x^2 - y^2}{(x^2 + y^2)^2}, \quad (x, y) \in I \times I.$$

Prove that

$$\int_I \left(\int_I h(x, y) dy \right) dx = \frac{\pi}{4},$$

$$\int_I \left(\int_I h(x, y) dx \right) dy = -\frac{\pi}{4}$$

and

$$\int_{I \times I} |h(x, y)| dx dy = \infty.$$

3. (1 p) Suppose f is a real-valued continuous function on the unit interval $[0, 1]$. Compute the limit

$$\lim_{n \rightarrow \infty} \int_0^1 f(x) n^2 x e^{-nx} dx.$$