INTEGRATION THEORY (2011)
(GU[$MMA110$] ,**CTH**[$tmv100$]**)**

ASSIGNMENT 1
(Must be handed in at the latest Tuesday at 11^{45}, week 38)
(18 p = 1 credit point)

1. (3 p) Let $(X, \mathcal{M}, \mu)$ be a positive measure space. Show that

$$\mu(\cap_{k=1}^{n} A_k) \leq \sqrt[n]{\Pi_{k=1}^{n} \mu(A_k)}$$

for all $A_1, ..., A_n \in \mathcal{M}$.

2. (3 p) Let $A \subseteq \mathbf{R}$ be Lebesgue measurable and $m(A) = 1$. Show that there exists a Lebesgue measurable set $E \subseteq A$ such that $m(E) = \frac{1}{2}$.

3. (3 p) Suppose μ is a σ-finite positive measure on $\mathcal{R}$. Prove that the set of all $x \in \mathbf{R}$ such that $\mu(\{x\}) > 0$ is at most denumerable.