INTEGRATION THEORY (2011)
(GU$[MAF110]$ **,CTH**$[tmv100]$**)**

ASSIGNMENT 2
Must be handed in at the latest Tuesday at 11^{45}, week 40.
(18 p = 1 credit point)

1. (3 p) Let $(X, \mathcal{M}, \mu)$ be a positive measure space and $f{:}X \to [0, \infty[$ an $(\mathcal{M}, \mathcal{R})$-measurable function such that $f(X) \subseteq \mathbf{N}$ and

$$\int_X f d\mu < \infty.$$

Prove that

$$\lim_{n\to\infty} nG(n) = 0$$

and

$$\int_X f d\mu = \Sigma_{n=1}^{\infty} G(n)$$

if $G(t) = \mu(f \geq t)$ for every $t \geq 0$.

2. (3 p) Compute

$$\lim_{n\to\infty} \int_0^1 \frac{1 + 2n^2x^4}{(1 + x^2)^n} dx.$$

3. (1+1+0.5+0.5 p) Let $(X, \mathcal{M}, \mu)$ be a positive measure space and suppose $f, g{:}X \to [0, \infty[$ are $(\mathcal{M}, \mathcal{R})$-measurable functions such that

$$\int_A f d\mu = \int_A g d\mu \text{ if } A \in \mathcal{M}.$$

(a) Prove that $f = g$ a.e. $[\mu]$ if $\mu(X) < \infty$, $\sup_X f < \infty$, and $\sup_X g < \infty$.
(b) Prove that $f = g$ a.e. $[\mu]$ if μ is finite.
(c) Prove that $f = g$ a.e. $[\mu]$ if μ is σ-finite.
(d) Prove that the conclusion in Part (c) may fail if μ is not σ-finite.