INTEGRATION THEORY (2011)
(GU$[MAF110]$ **,CTH**$[tmv100]$**)**

ASSIGNMENT 3
Must be handed in at the latest Tuesday at 11^{45}, week 42.
(18 p = 1 credit point)

1. (3 p) A Borel probability measure μ on $\mathbf{R}^n$ is given by the equation $d\mu = a\exp(-\mid x \mid^n)dx$, where a is a positive constant. (a) Find a. (b) Find the μ-measure of the Euclidean ball $B = \{x \in \mathbf{R}^n;\ \mid x \mid \leq 1\}$.

2. (3 p) Let $I =]0,1[$ and

$$h(x,y) = \frac{x^2-y^2}{(x^2+y^2)^2},\ \ (x,y) \in I \times I.$$

Prove that

$$\int_I (\int_I h(x,y)dy)dx = \frac{\pi}{4},$$
$$\int_I (\int_I h(x,y)dx)dy = -\frac{\pi}{4}$$

and

$$\int_{I\times I} \mid h(x,y) \mid dxdy = \infty.$$

3. (1+1+1 p) Suppose ν is a signed measure on $(X, \mathcal{M})$ and $E \in \mathcal{M}$. Prove that
   (a) $\mid \nu(E) \mid \leq \mid \nu \mid (E)$.
   (b) $\nu^+(E) = \sup\{\nu(F);\ F \subseteq E\}$.
   (c) $\mid \nu \mid (E) = \sup\{\Sigma_1^n \mid \nu(E_j) \mid;\ n \in \mathbf{N}_+,\ E_1, ..., E_n$ disjoint, and $\cup_1^n E_j = E\}$.