Let $m = \sup\{\mu(E_j) : E \in \mathcal{M}\}$. Take $A_n \in \mathcal{M}$ such that $m - \mu(A_n) < 1/2^n$.

Observe that in $B \in A_n$, then $\mu(B) \geq -1/2^n$, and if $C \in X \setminus A_n$ then $\mu(C) \leq 1/2^n$ (otherwise $\mu(A \setminus B) > m$ or $\mu(A \cup C) > m$).

Take $P = \cup_{n=1}^{\infty}(\cap_{k\geq n} A_k)$.

Let $B \subset P$. Then for every n, $B \subset \cap_{k\geq n} A_k$. Let $\tilde{A}_n = A_n \setminus (\cup_{j=1}^{n-1} A_j)$. By countable additivity, $\mu(B) = \sum_{k\geq n} \mu(B \cap \tilde{A}_k) > \sum_{k\geq n} -1/2^k = -1/2^{n+1}$, the last inequality due to $B \cap \tilde{A}_k \subset A_k$. As the estimate is true for every n we see that $\mu(B) \geq 0$, i.e. P is positive.

Let $C \subset X \setminus P$. Then $C \subset \cup_{n=1}^{\infty}(X \setminus \cup_{k\geq n} A_k)$. For every M holds

$$\cup_{n=1}^{\infty}(X \setminus \cup_{k\geq n} A_k) = \cup_{n=M}^{\infty}(X \setminus \cup_{k\geq n} A_k).$$

Set $N_n = X \setminus \cup_{k\geq n} A_k \subset X \setminus A_n$. It is easy to see that $\tilde{N}_n = N_n \setminus N_{n+1} \subset X \setminus A_n$. Thus, $\mu(C) = \mu(C \cap N_M) + \sum_{n=M}^{\infty} \mu(C \cup \tilde{N}_n) \leq 1/2^M + \sum_{n=M}^{\infty} 1/2^n = 3/2^M$. As M is arbitrary, this implies $\mu(C) \leq 0$, i.e. $X \setminus P$ is negative.