We choose $\{n_k\}$ so that $\mu(\{x : |f_{n_k}(x) - f^*(x)| \geq 1/2^k\}) < 1/2^k$.

Now consider $E = \{x : \exists k^* \forall N \exists k > N |f_{n_k}(x) - f^*(x)| > 1/2^{k^*}\}$, i.e. the set of x for which $\{f_{n_k}(x)\}$ does not converge to $f^*(x)$. Also, (check it)

$$E = \{x : \exists k^* \forall N > k^* \exists k > N |f_{n_k}(x) - f^*(x)| > 1/2^{k^*}\} \subset$$
$$\{x : \exists k^* \forall N > k^* \exists k > N |f_{n_k}(x) - f^*(x)| > 1/2^k\}$$

(the last as $k^* < k$).

Again

$$\{x : \exists k^* \forall N > k^* \exists k > N |f_{n_k}(x) - f^*(x)| > 1/2^k\} = \{x : \exists k^* \forall N \exists k > N |f_{n_k}(x) - f^*(x)| > 1/2^k\}$$
$$= \{x : \forall N \exists k > N |f_{n_k}(x) - f^*(x)| > 1/2^k\}$$

(the last as k^* was not involved in anything defining the set).

$$\{x : \forall N \exists k > N |f_{n_k}(x) - f^*(x)| > 1/2^k\} = \bigcap_N (\bigcup_{k>N} \{x : |f_{n_k}(x) - f^*(x)| > 1/2^k\}.$$

Thus,

$$\mu(E) \leq \mu(\{x : \forall N \exists k > N |f_{n_k}(x) - f^*(x)| > 1/2^k\}) =$$
$$\mu(\bigcap_N (\bigcup_{k>N} \{x : |f_{n_k}(x) - f^*(x)| > 1/2^k\}) \leq \lim_N \sum_{k>N} \mu(\{x : |f_{n_k}(x) - f^*(x)| > 1/2^k\})$$
$$\leq \lim_N \sum_{k>N} 1/2^k = \lim_N 1/2^{N+1} = 0.$$