General knowledge (to know at all times)

- Definition of σ-algebra

- Definition of a measure

- Definition of Borel σ-algebra/measure

- Formulation of Caratheodory Extension theorem

- Definition of measurable function

- Definition of integral of a positive, measurable function (any one)

- Definition of pointwise, a.e.-pointwise, L^1, and in measure convergency (with respect to a given measure μ)

- Formulation of Lebesgue-Radon-Nikodym decomposition

Theoretical questions

1. Construct a set which can not be measured by a shift invariant countably additive measure

2. Define π and d-systems and their connection with σ-algebras

3. Prove Caratheodory Extension theorem

4. Prove Monotone Convergence theorem

5. Prove Lebesgue Dominant Convergence theorem

6. Introduce integral for an L^1-function and show correctness of the definition/ additivity of integral (depending form which definition you take)

7. Describe connection between a.e.-pointwise and L^1 convergencies (with proofs)

8. Describe connection between a.e.-pointwise and in measure convergencies (with proofs)

9. Define product of measures and prove Fubini-Tonelli theorem

10. Prove two Borell-Cantelly theorems

11. Prove Kolmogorov 0-1 law

12. Prove Jordan-Hahn decomposition theorem

13. Prove Lebesgue-Radon-Nikodym theorem

14. Prove weak-boundedness of Hardy-Littlewood maximal function

15. Prove Lebesgue differentiation theorem

16. Define functions of bounded variation and describe their continuity properties (with proofs)

17. Formulate and prove Fundamental theorem of Calculus and integration by parts for Lebesgue integral

18. Define Riesz products and discuss their mutual singularity