

General knowledge (to know at all times)

- Definition of σ -algebra
- Definition of a measure
- Definition of Borel σ -algebra/measure
- Formulation of Caratheodory Extension theorem
- Definition of measurable function
- Definition of integral of a positive, measurable function (any one)
- Definition of pointwise, a.e.-pointwise, L^1 , and in measure convergency (with respect to a given measure μ)
- Formulation of Lebesgue-Radon-Nikodym decomposition

Theoretical questions

1. Construct a set which can not be measured by a shift invariant countably additive measure
2. Define π and d -systems and their connection with σ -algebras
3. Prove Caratheodory Extension theorem
4. Prove Monotone Convergence theorem
5. Prove Lebesgue Dominant Convergence theorem
6. Introduce integral for an L^1 -function and show correctness of the definition/ additivity of integral (depending from which definition you take)
7. Describe connection between a.e.-pointwise and L^1 convergencies (with proofs)
8. Describe connection between a.e.-pointwise and in measure convergencies (with proofs)
9. Define product of measures and prove Fubini-Tonelli theorem

10. Prove two Borell-Cantelly theorems
11. Prove Kolmogorov 0-1 law
12. Prove Jordan-Hahn decomposition theorem
13. Prove Radon-Nikodym part in Lebesgue-Radon-Nikodym theorem
14. Prove Lebesgue part in Lebesgue-Radon-Nikodym theorem
15. Prove weak-boundedness of Hardy-Littlewood maximal function
16. Prove Lebesgue differentiation theorem for absolutely continuous measure
17. Prove Lebesgue differentiation theorem for singular measure
18. Define functions of bounded variation and describe their continuity properties (with proofs)
19. Formulate and prove Fundamental theorem of Calculus and integration by parts for Lebesgue integral