QUANTUM GROUPS, QUANTUM SYMMETRIC SPACES AND OPERATOR ALGEBRAS

to the memory of Leonid Vaksman

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Programme of Talks

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Main Talks

Algebraic geometry of the classical Yang-Baxter equation

Igor Burban, University of Paderborn, Germany

In my talk, based on a joint work with Lennart Galinat, an algebro-geometric description of solutions of the classical Yang-Baxter equation (CYBE) will be explained. Namely, I am going to show that any pair (E, A), where E is an irreducible plane cubic curve and A a coherent torsion free sheaf of Lie algebras (whose generic fiber is a given complex simple Lie algebra) with vanishing cohomology, canonically defines a solution of CYBE. It turns out that at least all elliptic and rational solutions of CYBE arise this way. The developed method will be illustrated by explicit examples.

Peter-Weyl Bases, Preferred Deformations and Schur-Weyl Duality

Anthony Giaquinto, Loyola University Chicago, USA

We discuss the quantized function Hopf algebra of a simply connected reductive Lie group G over C using a basis consisting of matrix elements of finite dimensional representations. This leads to a preferred deformation in that basis, meaning one where the structure constants of comultiplication are unchanged. The structure constants of multiplication are controlled by quantum 3j symbols. We then discuss connections earlier work on preferred deformations that involved Schur-Weyl duality.

Quantized Differential Operators, Quantized Weyl Algebras, and Quantized Generalized Verma Modules

Hans Plesner Jakobsen, University of Copenhagen

The three topics mentioned in the title are intricately connected through what one may call the quantized harmonic representation. Classically, the proof of the Kashiwara-Vergne conjecture for the (tensor products of) the Metaplectic representation was shown to be connected to the covariance of important differential operators from physics (generalized Maxwell's equations). Dually, these covariant differential operators define homomorphisms between generalized Verma modules. The infinitesimal version of the K-V conjecture was never thoroughly discussed, but for the quantized case, T. Hyashi has introduced a family of important algebras; the quantized Weyl algebras $\mathcal{W}(n)$, and thereby obtained quantizations of many important representations.

Moving into the "q-world" in connection with Hermitian symmetric spaces, an important new fact of life is that polynomial algebras are replaced by quadratic algebras. This implies that multiplication operators, and dually, differential operators, need to have a side (left/right) attached. We will discuss algebras of quantized differential operators where we allow variable coefficients, and will see how $\mathcal{W}(n)$ and certain subalgebras appear naturally.

Finally, we will show how, on the one side, homomorphisms between quantized generalized Verma modules, and, on the other side, $\mathcal{W}(n)$, are intricately related.

Belavin-Drinfeld cohomology and classification of quantum groups

Eugene Karolinsky, Kharkiv National University, Ukraine

We introduce a cohomology-type theory for the classification of quasitriangular Lie bialgebra structures on split semisimple Lie algebras over $\mathbb{C}((t))$, and relate it to Galois cohomology. Further, basing on the now classical results of Etingof and Kazhdan, we apply this theory towards the classification of quantum groups with a given quasiclassical limit.

On compact quantum groups of Lie type

Sergey Neshveyev, University of Oslo, Norway

Given a compact connected Lie group G, we say that a compact quantum group is of G-type if its representation semiring together with the classical dimension function is isomorphic to that of G. The well-known examples of such quantum groups are the Drinfeld-Jimbo q-deformations of G, which on the function algebra side were first studied by Soibelman-Vaksman and Woronowicz in the case of G=SU(2). In my talk I will give some less known related examples and report on the progress in classification of such quantum groups. (Based on a series of joint papers with Makoto Yamashita.)

Regular Operators and Graph Regular Operators on Hilbert C^*-modules

Konrad Schmüdgen, Universität Leipzig, Germany

Let A be a C^*-algebra. A Hilbert A-module is a right A-module E which is equipped with an A-valued scalar product and complete in the corresponding norm. An operator between Hilbert A-modules E and F is an A-linear and $\mathbb{C}$-linear mapping $t : E \mapsto F$. Regular operators were introduced by S. Baaj (1981) and studied in a different setting (as affiliated operators on C^*-algebras) by S.L. Woronowicz (1991). Graph regular operators are a new class of operators invented by R. Gebhardt and the author (2014). In the talk regular operators will be reviewed and graph regular operators will be discussed in detail.

Assume that $t : E \mapsto F$ is an operator which is essentially defined, that is, $\mathcal{D}(t)^{\perp} = \{0\}$. Then t has a well-defined adjoint operator. An essentially defined operator t is called *graph regular* if its graph $\mathcal{G}(t)$ is orthogonally complemented in $E \oplus F$ and *orthogonally closed* if $\mathcal{G}(t)^{\perp\perp} = \mathcal{G}(t)$. These classes of operators are investigated. Various characterizations of graph regular operators are given. A number of examples of graph regular operators that are not regular are presented ($C_0(X)$, Toeplitz algebra, Heisenberg group).

Cohomological Hall algebras, quantum groups and instantons.

Yan Soibelman, Kansas State University, USA

Abstract. The notion of Cohomological Hall algebra (COHA for short) was introduced in 2010 in my joint paper with Maxim Kontsevich. In a special case of a quiver with potential it is given by the cohomology of the stack of representations of the quiver with coefficients in the sheaf of vanishing cycles of the potential. I am going to explain basic definitions and few examples when COHA can be computed. I will explain that in the case of the Jordan quiver (one vertex and one loop) the equivariant version of (spherical) COHA is isomorphic to the positive part of the affine Yangian for gl(1). Then,if time permits, I plan to discuss an application of this result to the recent work of physicists Gaitto and Rapcak on the so-called " vertex algebra at the corner", as well as further generalizations to more general quivers and Calabi-Yau threefolds.

On noncommutative fiber bundles

Wojciech Szymanski, South Denmark University, Denmark

The concepts of principal and associated vector bundles, so crucial to classical Differential Geometry and Topology, have been successfully incorporated into Noncommutative Geometry. However, it is less clear how to interpret the notion of a general (locally trivial) fibration in noncommutative setting. In this talk, I would like to present some initial thoughts and observations aimed at development of such a notion. The discussion will be motivated by concrete examples, including a quantum flag manifold and a quantum twistor bundle. This talk is based on an ongoing joint project with Tomasz Brzeziński and Sophie Emma Mikkelsen. The talk will be a survey of the theory of mapping cones of positive maps, their dual cones and Choi matrices for maps. I shall also comment on the relation to the theory of operator systems.

Geometry and Analysis on Jordan-Kepler Varieties

Harald Upmeier, University of Marburg, Germany

Hermitian symmetric domains D=G/K, of classical and exceptional type, contain interesting K-invariant algebraic varieties called Kepler varieties, which are defined by determinantal equations. In joint work with M. Englis (Prague), and also Gadadhar Misra (Bangalore), we study the geometric properties of Kepler varieties and their quantization via reproducing kernel Hilbert spaces. The main results are 1. Kepler varieties are normal varieties in the sense of algebraic geometry 2. The singular set of a Kepler variety has a resolution via a higher rank version of the classical blow-up process, with exceptional fibres given by compact hermitian symmetric spaces of higher rank 3. For a natural choice of invariant measure on Kepler varieties, the reproducing kernel of the associated Hilbert space of holomorphic functions can be expressed as a multi-variable hypergeometric series, generalizing the Faraut-Koranyi formula 4. There is a Rigiditiy Theorem for "truncated" Hilbert submodules vanishing on the singular set, using the generalized blow-up process. The underlying determinant equations and the geometric setting within flag manifolds (Matsuki duality) may also have interesting q-deformations.

Contributed Talks

The elliptic Lie Bialgebra in the algebro-geometric context

Raschid Abedin, University of Paderborn, Germany

Abstract: In the algebro-geometric approach to the CYBE each geometric r-matrix gives rise to a Lie bialgebra. We consider the classical elliptic r-matrices in the geometric context and describe the associated Lie bialgebras and coordinate expressions of their defining structures.

q-independence of the Drinfeld-Jimbo quantization

Olof Giselsson, Chalmers University of Technology and University of Gothenburg, Sweden

It has long been known that quantum $SU_q(2)$ is independent, as a C*-algebra, of the parameter $0 < q < 1$. In the 90's, it was shown by G. Nagy, that the same holds for $SU_q(3)$. By extending Nagy's methods, we show that for a general (simply connected, connected) compact Lie group G, the universal enveloping C^*-algebra of the quantized algebra of continuous functions on G is independent of q. This is proved using induction on the dimensions of symplectic leaves of G.

TBA

Jonathan Nilsson, Chalmers University of Technology and University of Gothenburg, Sweden

On CAR relations with orthogonal ranges

Vasyl Ostrovskyi, Institute of Mathematics, NAS of Ukraine

We consider $*$-representations of algebra with three generators a_1, a_2, a_3 satisfying the relations

$$\begin{aligned} \{a_j, a_j^*\} = I, \quad a_j^2 = 0, \qquad j = 0, 1, 2, \\ a_j^* a_k = 0, \qquad j \neq k. \end{aligned}$$

For this algebra, we show that the category of its $*$-representations is equivalent to the one of the algebra generated by elements A_1, A_2, A_1, A_2 where B_1 and B_2 satisfy the Cuntz-Toeplitz relations $B_j^* B_k = \delta_{jk} I$, $j, k = 1, 2$, and the following set of relations holds

$$\begin{aligned} A_1^* A_2 = 0, \quad \{A_j, A_j^*\} = I - B_j B_j^*, \quad A_j^2 = 0, \qquad j = 1, 2, \\ B_1^* A_1 = B_1^* A_1^* = B_2^* A_1 = 0, \quad B_2^* A_2 = B_2^* A_2^* = B_1^* A_2 = 0. \end{aligned}$$

We describe the Fock representation of the latter algebra, constract a series of irreducible non-Fock representations and give a commutative model for all representations for which $B_1B_1^* + B_2B_2^* + A_1A_1^* + A_2A_2^* = I$. Joint work with Daniil Proskurin, Kyiv National Taras Shevchenko University, Ukraine.

On representations of finite W-algebras

Elena Poletaeva, University of Texas Rio Grande Valley, USA

A finite W-algebra is a certain associative algebra W attached to a pair $(\mathfrak{g}, e)$, where $\mathfrak{g}$ is a complex semisimple Lie algebra and $e \in \mathfrak{g}$ is a nilpotent element. In the case when $\mathfrak{g}$ is the general linear Lie superalgebra $\mathfrak{gl}(m|n)$ and e is the even principal nilpotent, Brown, Brundan and Goodwin classified irreducible representations of W and explored the connection with the category $\mathcal{O}$ for $\mathfrak{g}$ using coinvariants functor. We study representations of W for the superalgebra $Q(n)$ associated with the principal even nilpotent coadjoint orbit. We obtain a classification of simple W-modules.

On q-tensor product of Cuntz algebras

Daniil Proskurin, Kyiv National Taras Shevchenko University, Ukraine

We consider the C^*-algebra $\mathcal{E}^q_{n,m}$, which is a q-twist of two Cuntz-Toeplitz algebras. For the case $|q| < 1$, we give an explicit formula which untwists the q-deformation, thus showing that the isomorphism class of $\mathcal{E}^q_{n,m}$ does not depend on q. For the case $|q| = 1$, we give an explicit description of all ideals in $\mathcal{E}^q_{n,m}$. In particular, we show that $\mathcal{E}^q_{n,m}$ contains a unique largest ideal $\mathcal{M}_q$. Then we identify $\mathcal{E}^q_{n,m}/\mathcal{M}_q$ with the Rieffel deformation of $\mathcal{O}_n \otimes \mathcal{O}_m$ and use a K-theoretical argument to show that the isomorphism class does not depend on q. Joint work with Alexey Kuzmin, Chalmers University of Technology and University of Gothenburg, Sweden, Vasyl Ostrovskyi, Institute of Mathematics, NAS of Ukraine, Roman Yakymiv Kyiv National Taras Shevchenko University, Ukraine.

TBA

Alexander Stolin, Chalmers University of Technology and University of Gothenburg, Sweden

Shilov boundary for "holomorphic functions" on a quantum matrix ball

Lyudmila Turowska, Chalmers University of Technology and University of Gothenburg, Sweden

The Shilov boundary of a compact Hausdorff space X relative to a uniform algebra $\mathcal{A}$ in $C(X)$ is the smallest closed subset $K \subset X$ such that every function in $\mathcal{A}$ achieves its maximum modulus on K. This notion is encountered, in particular, in the theory of analytic functions in relation to the maximum modules principle. We will be interested in its non-commutative analog. The latter was introduced by W. Arveson.

In the middle of 90s, within the framework of the quantum group theory, L.Vaksman and his coauthors started a "quantisation" of bounded symmetric domains. One of the simplest of such domains is the matrix ball $\mathbb{D} = \{z \in Mat_m : zz^* < I\}$, where Mat_m is the algebra of complex $m \times m$ matrices. The Shilov boundary of $\mathbb{D}$ relative to the algebra of holomorphic functions in $C(\overline{\mathbb{D}})$ is the set of unitary $m \times m$-matrices. In this talk I will discuss the Shilov boundary ideal for the q-analog of holomorphic functions on the unit ball. This is a joint work with O.Bershtein and O.Giselsson.

On isometries satisfying deformed commutation relations

Roman Yakymiv, Kiev National Taras Shevchenko University, Ukraine

We consider C^*-algebra $\mathcal{E}^q_{1,n}$, $q \leq 1$, generated by isometries satisfying q-deformed commutation relations. For the case $|q| < 1$ we prove that $\mathcal{E}^q_{1,n} \simeq \mathcal{E}^0_{1,n} = \mathcal{O}^0_{n+1}$. For $|q| = 1$ we show that $\mathcal{E}^q_{1,n}$ is nuclear, prove that its Fock representation is faithul. In this case we also discuss the rerpesentation theory, in particular construct the commutative model for representations. Joint work with Olha Ostrovska, National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"

Eigenvalues of invariant differential operators and Okounkov polynomials.

Genkai Zhang, Chalmers University of Technology and University of Gothenburg, Sweden

Okounkov introduced a class of Weyl-group invariant polynomials and their q-analogues as generalization of the Newton interpolation polynomials in one variable. We prove that Okounkov polynomials for root system of type BC are the eigenvalues of Shimura invariant differential operators and we study then the question of Shimura on the positivity of eigenvalues. (Joint work with S. Sahi.)