Tentamensskrivning i linjär och multilinjär algebra

MMA 200

Tisdagen den 20 oktober, 2009

V

8.30 - 13.30

1 Diagonalize the quadratic form

$$6x^2 + 6y^2 + 6z^2 + 4xz - 8yz$$

over the rationals.

2 Let M be an abelian group (i.e a $\mathbb{Z}$ module) with 144 elements.
a) Make a list of the possible non-isomorphic cases.
b) Find sharp upper and lower bounds for the cardinality of $\mathrm{End}(M)$

3 Compute the number of elements of the finite group $SO(3, \mathbb{F}_q)$ where F_q is a finite field with q-element (q odd to play it safe).
Hint: Exploit the parametrization $M \mapsto \frac{I+M}{I-M}$ where M is a skew-symmetric matrix. And make a separate analysis for the cases not covered by the parametrization.

4 Compute (up to conjugation) the number of ways the group S_3 can be seen as a subgroup of $GL(n, \mathbb{C})$ and $SL(n, \mathbb{C})$ respectively, recalling that the later group refers to those with determinant one.

5 Let A, B be two quadratic matrices. Compute the trace of the tensor product $A \otimes B$ of the two in terms of the traces of A and B

6 Let A be an integral 2×2 matrix with non-vanishing determinant. Consider the sub-module $\Lambda \subset \mathbb{Z}^2$ generated by its column vectors.
a) Give a necessary and sufficient condition on the matrix A such that the torsion-module $\mathbb{Z}^2/\Lambda$ is cyclic
b) Give an explicit example of a matrix A such that the quotient is not cyclic.
c) Is it possible to give an example of A such that the quotient is cyclic, but no vector of form $(0, a)$ or $(b, 0)$ is a generator?

7 Consider the subset G av $SO(4, \mathbb{R})$ consisting of matrices with integer entries.
a) Show that G is in fact a finite subgroup and determine its number of elements.
b) Present G as a subgroup of index two of a semi-direct product of two groups.
c) Compute the character of G of the given representation and show that it is irreducible.

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