Exercises

October 11, 2011
due
October 21, 2011

1 Let G be a simple group, i.e. a non-cyclic group with no non-trivial normal subgroups. Show

a) That every 1-dimensional representation is trivial

b) That every non-trivial representation is faithful. (I.e the group is represented as a subgroup of the general linear group.)

2 Let W be the irreducible 2-dimensional representation of S_3. Decompose the module $\mathrm{Hom}(W, W)$ into irreducible components.

3 Consider the finite group $SO(3, \mathbb{Z})$. Show that it is isomorphic to S_4

4 The following is the character-table for S_4

dimension	1	(12)	(12)(34)	(123)	(1234)
1	1	1	1	1	1
1	1	-1	1	1	-1
2	2	0	2	-1	0
3	3	1	-1	0	-1
3	3	-1	-1	0	1

a) The previous exercise gives rise to a 3-dimensional representation of S_4. Which one?

b) S_2 and D_8 are two subgroups of S_4. Find the corresponding decompositions into irreducible components of the first 3-dimensional irreducible representation in the table.

5 The group $SO(4, \mathbb{Z})$ represent the orientation preserving automorphisms of the 4-dimensional hypercube.

a) Compute its order and the number of conjugacy classes!

b) Is it possible to find a homomorphism $SO(4, \mathbb{Z}) \to S_4$?

c) Would you be able to find its complete character table?

6 The symmetric group S_n operates on $\mathbb{C}^n$ by permutation of its co-ordinates. Try and write down its character! Is it an irreducible character? Can you decompose it into explicit irreducible characters?