Advanced linear and multilinear algebra (MMA200)

Time: 2015-10-26, 8:30–12:30.

Tools: No calculator or handbook is allowed.

Questions: Hjalmar Rosengren, 031-7725358

Grades: Each problem gives 6 points. At most 6 bonus points from the exercise sessions will be added to the result. Grades are G (15-24 points) and VG (25-30).

---

1 Compute the number of abelian groups of order 144. Write down explicitly all such groups G such that $G/3G \simeq \mathbb{Z}/3\mathbb{Z}$ and $2G/4G \simeq \mathbb{Z}/2\mathbb{Z}$.

2 Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Find the following canonical forms of A: The Jordan canonical form, the primary canonical form over $\mathbb{Q}$ and the rational canonical form.

3 Let $(e_j)_{j=1}^3$ be the standard basis of $\mathbb{R}^3$. Show that the elements $u_1 = e_1 \wedge (e_2 + e_3)$, $u_2 = e_2 \wedge (e_3 - e_1)$ and $u_3 = (e_1 + e_2) \wedge e_3$ form a basis for $\mathbb{R}^3 \wedge \mathbb{R}^3$. If $A \in \mathrm{End}(\mathbb{R}^3)$ is given in the standard basis by

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

compute the matrix for $A \wedge A$ in the basis u_1, u_2, u_3.

4 Define irreducible elements and prime elements in an integral domain. Show that in a PID, every irreducible element is prime.

5 Let V and W be irreducible representations of a group G, with characters χ_V and χ_W. Prove that

$$\frac{1}{|G|} \sum_{g \in G} \chi_V(gh)\overline{\chi_W(g)} = \begin{cases} \dfrac{\chi_V(h)}{\dim V}, & V \simeq W, \\ 0, & \text{else.} \end{cases}$$

(It is assumed that G is finite and that V and W are finite-dimensional complex vector spaces.)