Advanced linear and multilinear algebra (MMA200)

Time: 2016-01-07, 14:00-18:00.

Tools: No calculator or handbook is allowed.

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Grades: Each problem gives 6 points. At most 6 bonus points from the exercise sessions will be added to the result. Grades are G (15-24 points) and VG (25-30).

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1 Find all abelian groups of order 162. Give both the primary decomposition and the invariant factor decomposition of each group.

2 Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Find the Jordan canonical form and the minimal polynomial of A.

3 Let $v_1, \ldots, v_m$ be linearly independent elements of a finite-dimensional vector space and let $u \in \bigwedge^k V$ be such that $u \wedge v_j = 0$ for each $j = 1, \ldots, m$. Prove that $k \geq m$ and that $u = v_1 \wedge \cdots \wedge v_m \wedge w$ for some $w \in \bigwedge^{k-m} V$.

4 Prove that if V is a complex representation of a finite group G, and W is a subrepresentation of V, then there exists a subrepresentation W' with $V = W \oplus W'$.

5 A finite group G has six conjugacy classes, which we denote $C_1, \ldots, C_6$. We know two irreducible characters χ_1 and χ_2 of the group, given by

	C_1	C_2	C_3	C_4	C_5	C_6
χ_1	1	-1	1	-1	i	$-$i
χ_2	2	2	-1	-1	0	0

Compute the whole character table of G. Also determine the number of commutators in G, that is, the number of elements of the form $aba^{-1}b^{-1}$, with a, $b \in G$.