Higher differential calculus (MMA210)

- Study guide for the exam -

All numbers refer to the book by Madsen and Tornehave. Starred items are to be considered particularly important. Note however that this does **not** *necessarily mean that only these may appear on the exam.*

1 Preliminaries (chapter 1)

- *Example 1.2
- Definition 1.3
- *Theorem 1.4 (statement and proof)
- *Theorem 1.5 (statement and proof)
- *Theorem 1.6 (statement and proof)
- Example 1.7
- Example 1.8

2 Alternating algebra

- *Definition 2.1
- Lemma 2.2 (statement and proof)
- Example 2.3
- Definition 2.4
- *Definition 2.5
- *Lemmas 2.6-2.9 (statement)
- *Lemma 2.13 (statement and proof)
- Lemma 2.14 (statement)

- *Theorem 2.14 (statement and proof). Note that here you can follow the proof given in the book or the proof given in my notes (see the course homepage), using multi-indices.

3 Differential forms

- *Definition 3.1
- *Understand the notion of derivative of a smooth map (p. 15).
- *Definition 3.2
- *Example 3.3
- *Lemmas 3.4-3.6 (statement and proof)
- *Theorem 3.7 (statement)

4 De Rham cohomology

- *Definition 3.8
- Lemma 3.9 (statement)
- *Definition 3.10 (pullback)
- *Example 3.11
- *Theorem 3.12 (statement)
- *Example: Calculate the de Rham cohomology of $\mathbb{R}^2 - \{0\}$
- *Theorem 3.15 [Poincaré lemma] (statement and proof)

5 Smooth manifolds

- Definition 8.1
- *Definition 8.3

- *Definition 8.4
- *Understand notion of smooth atlas and differentiable structure.
- *C^∞-compatilibility of atlases
- *Local coordinate parametrization of a point $p \in \mathcal{M}$
- *Understand notion of *dimension* of a manifold
- *Diffeomorphism between smooth manifolds
- *Examples of manifolds: Euclidean space, open subsets $V \subset \mathcal{M}$, graph of a smooth function, general linear group $GL(n, \mathbb{R})$, n-sphere S^n.
- *Product manifolds (e.g. tori $T^n = S^1 \times \cdots \times S^1$)
- Quotient manifolds (e.g. circle, $S^1 \cong I/\sim$, real projective space $\mathbb{R}P^n$)
- Submanifolds (Definition 8.8)
- Embedding (Definition 8.10)

6 Differential forms on manifolds

- *The tangent space $T_p\mathcal{M}$ of a manifold: Remark 9.1 and Definition 9.2
- The notion of *derivation* and relation with the tangent space
- Remark 9.4 (basis for the tangent space)
- *Directional derivative
- *Definition 9.5 (smooth differential form)
- Lemma 9.6 (statement and proof)
- *Exterior differential on a manifold
- *Orientation of a manifold (Definition 9.8)
- *Orientation-preserving diffeomorphism
- *Oriented atlas
- *Partition of unity (Thm 9.11, only statement)

7 de Rham cohomology of a manifold

- *Definition 9.7
- *The Mayer-Vietoris sequence (Thm 5.2)
- *Example: de Rham cohomology of a sphere S^n.
- *Example: de Rham cohomology of the punctured plane $\mathbb{R}^2 - \{0\}$.
- *Example: de Rham cohomology of a torus T^2.

8 Integration on manifolds

- *Lemma 10.1 (statement and proof)
- *Proposition 10.2 (statement and proof)
- *Lemma 10.3 (statement and proof)
- *Definition of a manifold with boundary
- *Stokes' theorem for an oriented manifold with boundary (statement and proof)