MMA320 Introduction to Algebraic Geometry

Exercises and Home work V 3

Exercises

1. Dolgachev 1.1, 1.2, 1.3, 1.5, 2.1.

2. Let K be an algebraically closed field extension of k. Let I be an ideal in $k[T]$ and let

$$V(I) = \{x \in K^n \mid F(x) = 0 \text{ for all } F \in I\} .$$

Show the following:
a) $V(0) = \mathbb{A}^n_k(K)$, $V(k[T]) = \emptyset$,
b) $I \subset J$ implies that $V(I) \supset V(J)$,
c) $V(I \cap J) = V(I) \cup V(J)$,
d) $V(\sum_{\lambda\in\Lambda} I_\lambda) = \cap_{\lambda\in\Lambda})V(I_\lambda)$.

3. Dolgachev 2.2, 1.4.

Home work, to be handed in 2010-01-27.

1. a) Which of the following rings are noetherian:
i) the polynomial ring $k[T_1, T_2, \dots, T_n, \dots]$ in infinitely many variables T_1, T_2, $\dots$, T_n,$\dots$
ii) the ring of all power series $a_0 + a_1 z + a_2 z^2 + \dots$ with positive radius of convergence, with all $a_i \in \mathbb{C}$.
b) Is A a noetherian ring if $A[x]$ is noetherian?

2. a) Let A be a noetherian ring and I an ideal. Show that the quotient ring $B = A/I$ is noetherian.
b) Let A be a noetherian integral domain and $A \subset K$ its field of fractions; let $0 \notin S \subset A$ be a subset and set

$$B = A[S^{-1}] = \{a/b \in K \mid a \in A, \text{ and } b = 1 \text{ or a product of elements of } S\} .$$

Show that B is noetherian.

3. a) Dolgachev 2.3 and 2.5
b) Dolgachev 2.4 and 2.6