MMA320 Introduction to Algebraic Geometry

Exercises and Home work V 4

Exercises

1. Dolgachev 2.1, 3.1, 3.2, 3.2, 3.5.

2. Let K be an algebraically closed field of characteristic $p > 0$. Consider the Frobenius map $F\colon K^n \to K^n$ given by $F(x_1, \dots, x_n) = (x_1^p, \dots, x_n^p)$. Show that F is regular and bijective, but not an isomorphism.

Home work, to be handed in 2010-02-03.

1. Let K be an algebraically closed field extension of k. Let I be an ideal in $k[T, S] := k[T_1, \dots, T_n, S_1, \dots, S_m]$. By example 3.3.3 the inclusion $k[T] \subset k[T, S]$ defines the projection $p\colon K^{n+m} \to K^n$. Let $J = I \cap k[T]$. Show that $V(J)$ is the Zariski closure of the image $p(V(I))$ of $V(I)$ under the projection p.

2. Prove that a morphism $f\colon X \to Y$ between two affine algebraic varieties is an isomorphism of X with a subvariety $f(X) \subset Y$ if and only if the induced map $f^*\colon \mathcal{O}(Y) \to \mathcal{O}(X)$ is surjective.

3. Dolgachev 3.4 and 3.7