MMA320 Introduction to Algebraic Geometry

Exercises and Home work V 5

Exercises

1. Dolgachev 4.2, 4.3, 4.5, 5.6

2. a) Describe the curve $C_1\colon 2X + Y^2 = 1$ in the other two standard coordinate charts on $\mathbb{P}^2(\mathbb{C})$. Hint: first homogenise the equation with a coordinate Z.
b) Let C_2 be defined by the equation $Y = X^3$ in the affine chart $Z = 1$. How does C_2 look like at infinity? Give an equation and draw the real part.

3. **Duality**. Let $\{e_0, e_1, e_2\}$ be a basis of $V \cong \mathbb{R}^3$ and let $(X_0 : X_1 : X_2)$ be corresponding homogeneous coordinates on $\mathbb{P}(V) \cong \mathbb{P}^2(\mathbb{R})$. Let $\{e_0^*, e_1^*, e_2^*\}$ be the dual basis in V^*, and $(U_0 : U_1 : U_2)$ corresponding homogeneous coordinates on $\mathbb{P}(V^*) \cong \mathbb{P}^2(\mathbb{R})$. Let $p = (a_0 : a_1 : a_2)$ be a point in $\mathbb{P}(V)$. Describe the pencil of all lines in $\mathbb{P}(V)$ through p in the coordinates $(U_0 : U_1 : U_2)$.

Home work, to be handed in wednesday 2010-02-10.

1. Dolgachev 4.1 and 4.4

2. Let $k = \mathbb{Z}/(2)$ be the field with two elements. How many points has $\mathbb{P}^2(k)$? How many lines pass through $P = (1 : 0 : 0)$? How many points lie on each of these lines? Draw all points and lines. Hint: choose $(1 : 0 : 0)$, $(0 : 1 : 0)$ and $(0 : 0 : 1)$ as vertices in a triangle and $(1 : 1 : 1)$ as interior point.

3. Let V be an irreducible affine algebraic k-set. We say that a rational function $F \in R(V)$ is regular at a point $a \in V$ if there exists an expression $F = P/Q$ with $P, Q \in \mathcal{O}(V)$ and $Q(a) \neq 0$. The domain of definition of F is

$$\operatorname{dom} F = \{a \in V \mid F \text{ regular at } a\}\ .$$

Define the ideal of dominators as

$$D_F = \{Q \in \mathcal{O}(V) \mid QF \in \mathcal{O}(V)\}\ .$$

a) Show that $\operatorname{dom} F$ is open and dense in the Zariski topology.
b) Show that $\operatorname{dom} F = V$ if and only if $F \in \mathcal{O}(V)$.