

MMA320 Introduction to Algebraic Geometry
Exercises and Home work V 7

Exercises

1. Hand out on Bezout: exercises 5 – 7
2. Dolgachev 8.1, 8.4, 8.5, 8.7

Home work, to be handed in wednesday 2010-02-24.

1. Let X be the cubic curve in $\mathbb{P}^2(\mathbb{Q})$ with equation:

$$y^2 = x^3 - 2x^2 + 1 .$$

Take $(0 : 1 : 0)$, the inflection point at infinity, as neutral element of the group law. Compute all multiples $2p, 3p, 4p, 5p, \dots$, of the point $p = (0, 1)$.

2. The irreducible cubic curve $X:Y^2Z - X^3 = 0$ has a cusp in $q = (0 : 0 : 1)$ and an inflection point in $(0 : 1 : 0)$. Show that for any field K the set $X^{\text{ns}}(K) = X(K) \setminus q$ has a group structure isomorphic to the additive group K^+ of the field K .
Hint: take the inflection point as neutral element and use a suitable parametrisation of $X^{\text{ns}}(K)$.
3. Let K be an algebraically closed field. Show that $W = K^2 \setminus (0, 0)$ is not an affine algebraic set.
Hint: show that $\mathcal{O}(W) = K[X, Y]$. Then you can use proposition 3.1