MMA320 Introduction to Algebraic Geometry
Exercises and Home work V 8

Exercises

1. Find all 27 lines on Clebsch' diagonal surface:

$$\begin{aligned} T_0^3 + T_1^3 + T_2^3 + T_3^3 + T_4^3 &= 0 \\ T_0 + T_1 + T_2 + T_3 + T_4 &= 0 \end{aligned}$$

All lines are real, 15 are easy to see as intersections with coordinate hyperplanes.

2. Let l_1, l_2, l_3 and l_4 be disjoint lines in $\mathbb{P}^3(K)$. Either all four lines lie on a smooth quadric Q, and they have infinitely many common transversals (a transversal being a line intersecting another line), or they do not lie on any quadric and then they have one or two common transversals.

3. Dolgachev 12.2, 12.3, 12.6, 12.7

Home work, to be handed in wednesday 2010-03-03.

1. Prove that given three disjoint lines in $\mathbb{P}^3(K)$ (K an algebraically closed field of characteristic unequal to 2), there exists a nonsingular quadric containing them. Hint: Take three points on each line, conclude that there is at least one quadric through these nine points, by looking at the dimension of the linear system of quadrics. Then rule out that the quadric is singular.

2. Find the singular points of the cubic surface

$$T_0T_1T_2 + T_0T_1T_3 + T_0T_2T_3 + T_1T_2T_3 = 0 .$$

Determine all lines on the surface.

3. Prove that the intersection of a hypersurface $V(F) \subset \mathbb{A}^n$ (not a hyperplane) with the tangent hyperplane $T(V)_p$ in a nonsingular point $p \in V$ is singular at p. (Here one defines the intersection by the ideal (F, L), where L is the linear equation for the tangent hyperplane.)
Show that this need not be true if one defines the intersection with reduced structure, i.e., by the radical of the ideal (F, L).