MAN460: Theory check-list

Some questions on the exam will be of a theoretical nature. For instance, you can be asked to reproduce a definition, a theorem or the proof of a theorem. There can also be more subtle questions like adapting the idea behind a proof to a slightly different situation, explaining why the hypotheses in a theorem are necessary etc.

Everyone should be able to state all definitions and theorems; such questions are always considered as "easy".

The list given below is not intended to be complete. It also contains vague statements that are clarified in the literature. It is better to use it to check how much you have learned, rather than as a list of things to memorize.

The main theory can be found in the following chapters. Sections in parentheses can be considered as for orientation; they contain things you should have an idea about but proofs are not included.

Andersson-Böiers: 1.1–1.4, (1.5–1.6), 2.1–2.3, (2.4), 4.1–4.2, (4.3), 5.1–5.4, (5.5).

Teschl: 2.1-2.3, 2.4 (only Thm 2.8), 2.5 (skip Thm 2.16), (2.6), 3.1–3.2, 3.4, (3.6), Notes on Lyapunov's method, Notes on linearization (some basic definitions are found in 6.1 and 6.3), (8.1–8.3), Notes on boundary value problems.

1. Existence and uniqueness theorems for initial value problems with Lipschitz condition.
2. Theorem about maximal solutions eventually leaving compact sets.
3. Dimension of the space of solutions to a homogeneous linear system.
4. Solution of inhomogeneous linear system using the fundamental matrix.
5. Perturbation of initial value problem (Teschl Thm 2.8, AB Sats 1.8).
6. Peano's existence theorem: proof not included.
7. Definition and properties of the operator norm of a matrix.
8. Definition and properties of the matrix exponential function.
9. Cayley-Hamilton's theorem. How can it be used to compute e.g. the matrix exponential?
10. Classification of plane linear autonomous systems.
11. Basic properties of phase portraits for autonomous systems. Why does exactly one orbit pass through each point? What can a maximal orbit look like?
12. Definition of equilibrium point, stable, unstable and asymptotically stable.
13. Lyapunov's method for investigating equilibrium points (several theorems).
14. Investigating equilibrium points through linearization.
15. Poincaré–Bendixson's Theorem. Proof not included.
16. The connection between solvability of homogeneous and inhomogeneous boundary value problems.

17. Green's function for boundary value problems.

18. Definition of symmetric differential operator. Symmetry of the operator $(ay')' + by$ with separated boundary conditions.

19. Spectral theorem for symmetric operators: proof not included.