Department of Mathematical Sciences, Chalmers& University of Gothenburg

MMA421, TMA013
Ordinary differential equations and dynamical systems

2010–06–04 kl. 8.30–13.30

You may not bring any notes, books or any other aids, not even a calculator!
To pass the exam (*i.e.* to obtain the grade "G" for (MMA421, GU), or grade "3" (TMA013, Chalmers)), you need 15 points. The final grade on the course depends also on the computer assignments.
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1. Prove that if $f : \mathbb{R}^n \to \mathbb{R}^n$ is Lipschitz continuous, the the initial value problem

$$\begin{aligned} \dot{x} &= f(x) \\ x(0) &= x_0 \end{aligned} \tag{1}$$

has a unique solution for t belonging to a sufficiently small interval around 0. (5p)

2. Consider the differential equation (1) again, where $f : \mathbb{R}^n \to \mathbb{R}^n$ is such that (at least locally), the differential equation has a unique solution.

Define the *flow* of the vector field f, and explain its relation with solutions to equation (1). Be careful to state the domain of definition of the flow. (5p)

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3. Consider the second order equation $\ddot{y} = f(y)$ $(y \in \mathbb{R})$. Show that if $F'(y) = f(y)$, then

$$E(t) = \frac{\dot{y}(t)^2}{2} - F(y(t))$$

is constant along solutions to the differential equation. (5p)

4. Consider the differential equation $\dot{x} = \cos(x^2 + t^2)$. Rewrite this equation as an autonomous system and sketch its phase portrait. (5p)

5. Give the definition of the ω_+ limit set. For dynamical system given by a differential equation in $\mathbb{R}^n$, $\dot{x} = f(x)$, is it possible that $\omega_+(x)$ consists of exactly two points? Motivate your answer well. (5p)

6. The Lorentz system is

$$\begin{aligned} \dot{x} &= -ax + ay \\ \dot{y} &= rx - y - xz \\ \dot{z} &= -bz + xy \end{aligned}$$

The origin is always a critical point for this system. For which (positive) values of the parameters a, b, and r, is the origin asymptitically stable? Which are the remaining fixed points? Are they asymptotically stable for some (positive) values of the parameters? (5p)

Good luck!
Bernt W.