Department of Mathematical Sciences, Chalmers& University of Gothenburg

MMA421, TMA013
Ordinary differential equations and dynamical systems

2010–08–11 kl. 8.30–13.30

You may not bring any notes, books or any other aids, not even a calculator!
To pass the exam (*i.e.* to obtain the grade "G" for (MMA421, GU), or grade "3" (TMA013, Chalmers)), you need 15 points. The final grade on the course depends also on the computer assignments.
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1. The motion of a particle moving along a line is governed by the equation

$$\dot{x} = t^3 - x^3$$

If the particle is located at $x = 2$ when $t = 2.5$, can it reach $x = 1$ at any later time? Motivate your answer. (5p)

2. Sketch the phase plane of the system

$$\begin{aligned} \dot{x} &= x^2 - xy \\ \dot{y} &= 1 \end{aligned}$$

(5p)

3.
   - Explain what is meant by the *Jordan canonical form* of a matrix, and state a theorem concering the existence of such forms.
   - What does the Jordan form of the matrix for a linear system of differential equation tell you about the solutions the system? (5p)

4. This problem concerns the so-called Hartmann-Grobmann theorem.
   - Let f be a differentiable vector field, and assume that 0 is a fixed point. What does it mean to say that the fixed point is hyperbolic?
   - Let $\dot{x} = f(x)$ have a linearization $\dot{y} = Ay$. What does the Hartmann-Grobmann theorem state about the relation between the solutions $x(t)$ and $y(t)$

(5p)

5. A heteroclinic orbit is an orbit that starts in the unstable manifold of one fixed point x_0 and arrives at the stable manifold of a fixed point x_1. If $x_0 = x_1$, the orbit is called homoclinic. For each of the two systems below, find *either* a homoclinic orbit *or* a heteroclinic orbit.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ \sin(x) \end{pmatrix} \qquad \frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x + x^2 \end{pmatrix}$$

(5p)

Good luck!
Bernt W.