Department of Mathematical Sciences, Chalmers& University of Gothenburg

MMA421, TMA014
Ordinary differential equations and dynamical systems
2011–06–09 kl. 8.30–13.30

You may not bring any notes, books or any other aids, not even a calculator!
To pass the exam (*i.e.* to obtain the grade "G" for (MMA421, GU), or grade "3" (TMA014, Chalmers)), you need 15 points. The final grade on the course depends also on the computer assignments.
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1. State and prove a theorem concerning the global in time existence of solutions to odes with a right hand side that is growing at most linearly in x. Be careful to give all necessary definitions. (5p)

2. Let C be a closed subset of a Banach space X, and let $K : C \to C$ be a contraction. Explain what is meant by the word *contraction*, and state and prove the *contraction mapping principle*. (5p)

3. Show that the differential equation $\dot{x} = f(t)x + g(t)x^n$ can be transformed into a first order linear equation by setting $y = x^{1-n}$. Then write the solution to the equation in y. (5p)

4. Sketch the phase portrait of the system
$$\begin{aligned} \dot{x} &= x + y^2 \\ \dot{y} &= -y \end{aligned}$$
(5p)

5. Find the flow of the system given in the previous problem. Be careful to state the domain of definition. (5p)

6. Let $\phi(t, t_0, y)$ be the (unique) solution to the system
$$\begin{cases} \dot{x} = f(x,t) \\ x(t_0) = y \end{cases}$$
To study the the sensitivity of solutions with respect to changes in initial data, it is useful to write an equation for $\frac{\partial}{\partial y}\phi(t, t_0, y)$ (note that this is a matrix if $y \in R^d$ with $d > 1$). Write this equation. Is it linear? Homogeneous? Autonomous? What assumptions do you have to make on f if this is to make sense? (5p)

Good luck!
Bernt W.