1. **a)** $D(x\ln x) = 1\cdot\ln x + x\frac{1}{x} = 1+\ln x$ **b)** $D(\frac{\tan x}{e^{3x^2}}) = \frac{(1+\tan^2 x)e^{3x^2}+(\tan x)6xe^{3x^2}}{(e^{3x^2})^2} = \frac{(\tan^2 x+6x\tan x+1)e^{3x^2}}{(e^{3x^2})^2} = \frac{\tan^2 x+6x\tan x+1}{e^{3x^2}}$ **c)** $D(\tan\sin x^2) = (1/\cos^2(\sin x^2))D(\sin x^2) = (1/\cos^2(\sin x^2))(\cos x^2)D(x^2) =$ $= (1/\cos^2(\sin x^2))(\cos x^2)2x = \frac{2x\cos x^2}{\cos^2(\sin x^2)}$ **d)** $D(x^{\cos x}) = D(e^{\ln(x^{\cos x})}) = D(e^{\cos x\ln(x)}) =$ $= e^{\cos x\ln(x)}D(\cos x\ln(x)) = x^{\cos x}(-\sin x\ln x + (\cos x)\frac{1}{x})$

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2. $\lim_{x\to-\frac{1}{2}^+} f(x) = (-\frac{1}{2})^2 = \frac{1}{4}$, $\lim_{x\to 1^-} f(x) = (1)^2 = 1$, $\lim_{x\to\frac{1}{2}^-} f(x) = -\frac{1}{2}a+b$, $\lim_{x\to 1^+} f(x) = (1)^2 = 1$

   Kontinuitet $\Rightarrow \begin{cases} -\frac{1}{2}a+b &= \frac{1}{4} \\ a+b &= 1 \end{cases} \Rightarrow a=b=1/2.$

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3. **a)** $0 = 4^{x+1} - 2\cdot 2^{x+2} + 4 = 4(2^x)^2 - 2(2^2)2^x + 4 \Rightarrow 0 = z^2-2z+1 = (z-1)^2$ där $z = 2^x$. Alltså har vi att $2^x = z = 1$ som ger att $x = 0$. **b)** Ekvationen $\Rightarrow x = (\sqrt{2x+1}-1)^2 = 1+2x+1-2\sqrt{2x+1} \Rightarrow$ $2\sqrt{2x+1} = x+2 \Rightarrow 4(2x+1) = x^2+4x+4 \Rightarrow x^{-}4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow x = 0$ eller $x = 4$. Endast $x = 0$ är en äkta rot; $x = 4$ har tillkommit vid kvadrering.

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4. **a)** $\lim_{x\to\infty}\frac{x^3(2-1/x+1/x^3)}{x^2(3+1/x-2/x^2)} = \lim_{x\to\infty} x\frac{2-1/x+1/x^3}{3+1/x-2/x^2} = \infty$ **b)** $\lim_{x\to\infty}\frac{\cancel{x}^3(1-1/x^2)(5+1/x)}{\cancel{x}^3(3+1/x-2/x^2)} = 5$ **c)** $\lim_{x\to\infty}\sqrt{x^2+1} - \sqrt{x^2-x+1} = \lim_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt{x^2-x+1}}{\sqrt{x^2+1}+\sqrt{x^2-x+1}}\frac{\sqrt{x^2+1}+\sqrt{x^2-x+1}}{1} = \lim_{x\to\infty}\frac{x^2+1-(x^2-x+1)}{\sqrt{x^2+1}+\sqrt{x^2-x+1}} =$ $= \lim_{x\to\infty}\frac{x}{|x|\sqrt{1+1/x^2}+|x|\sqrt{1-1/x+1/x^2}} = \lim_{x\to\infty}\frac{\cancel{x}}{\cancel{x}(\sqrt{1+1/x^2}+\sqrt{1-1/x+1/x^2})} = \frac{1}{1+1} = \frac{1}{2}$

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5. $0 = 8x^3-4x^2-6x+3 = (x-1/2)(8x^2+bx-6) =$ [$b = 0$ genom ihopmultiplikation och identifikation av koefficienter] $= (x-1/2)(8x^2-6) = (2x-1)(4x^2-3) = (2x-1)((2x)^2-3) \Rightarrow\ x = 1/2$ eller $x = \pm\sqrt{3}/2$.

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6. Sätt $y = \sin x \Rightarrow 2y^2+y-1 = 0 \Rightarrow y = -1$ eller $y = 1/2$. Två fall: i) $\sin x = -1 \Rightarrow x = \frac{3\pi}{2}+n2\pi$, ii) $\sin x = 1/2 \Rightarrow x = \frac{\pi}{6}+n2\pi$ eller $x = \pi-\frac{\pi}{6}+n2\pi = \frac{5\pi}{6}+n2\pi$.

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7. **a)** $\int_0^1\frac{1}{x^2-5x+6}\,dx = \int_0^1\frac{1}{(x-2)(x-3)}\,dx = \int_0^1\frac{A}{(x-2)}+\frac{B}{(x-3)}\,dx = \int_0^1\frac{-1}{(x-2)}+\frac{1}{(x-3)}\,dx = [-\ln|x-2|+\ln|x-3|]_0^1 =$ $-\ln 1+\ln 2-(-\ln 2+\ln 3) = 2\ln 2-\ln 3$ **b)** $\int\frac{\sin x}{1+\cos^2 x}\,dx = [t = \cos x,\ dt = -\sin x\,dx] = -\int\frac{1}{1+t^2}\,dt =$ $-\arctan t + C = -\arctan(\cos x) + C$

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8. Se kursboken för ett bevis av integralkalkylens huvudsats.
