MMG010 Introduktionskurs 090829

1 a) $\frac{3x}{4} + \frac{4}{3} = \frac{5}{6} + \frac{2x}{3} \Leftrightarrow \left(\frac{3}{4} - \frac{2}{3}\right)x = \frac{5}{6} - \frac{4}{3} \Leftrightarrow \frac{x}{12} = \frac{-3}{6} \Leftrightarrow x = -6$ b) $x+4 = 3x^2 \Leftrightarrow x^2 - \frac{x}{3} - \frac{4}{3} = 0 \Leftrightarrow$
$x_{1,2} = \frac{1}{6} \pm \sqrt{\frac{1}{36} + \frac{4}{3}} = \frac{1}{6} \pm \frac{7}{6} = -1, \ 4/3$ c) $5x + 6y = 7 \Leftrightarrow y = -\frac{5}{6}x + \frac{7}{6}$ Sökt linje: $k = \frac{-5}{6} = \frac{y-4}{x-3}$
$\Leftrightarrow 5x + 6y = 39$ d) $\ln 10 \lg e = \lg e^{\ln 10} = \lg 10 = 1$

2. a) Sätt $\sin x = z \Rightarrow z^2 + \frac{5}{2}z + 1 = 0 \Rightarrow z_{1,2} = -2, -1/2$. $z = -2$ omöjligt så vi
har $\sin x = z = -\frac{1}{2} \Rightarrow x = \begin{cases} -30^\circ + n360^\circ \\ 210^\circ + n360^\circ \end{cases}$ och om $0 < x < 360^\circ$ fås $x_1 = 210^\circ$, $x_2 = 330^\circ$

b) $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{2+3}{1-2\cdot 3} = -1$ så $\tan(u+v+w) = \frac{\tan(u+v) + \tan w}{1 - \tan(u+v)\tan w} = \frac{-1+4}{1-(-1)4} = \underline{\underline{\frac{3}{5}}}$

3) Vi har att $(\sqrt{1+4x})^2 = (1+\sqrt{4+x})^2 \Rightarrow \sqrt{1+4x} = |\sqrt{1+4x}| = \left((\sqrt{1+4x})^2\right)^{1/2} = \left((1+\sqrt{4+x})^2\right)^{1/2} = |1+\sqrt{4+x}|$
$= 1 + \sqrt{4+x}$. Alltså har vi $\sqrt{1+4x} = 1 + \sqrt{4+x} \Leftrightarrow (\sqrt{1+4x})^2 = (1+\sqrt{4+x})^2 \Leftrightarrow 1 + 4x = 1 + 4 + x + 2\sqrt{4+x}$
$\Leftrightarrow 3x - 4 = 2\sqrt{4+x} \Rightarrow$ (ej ekvivalens !!) $9x^2 - 24x + 16 = 4(4+x) \Leftrightarrow 9x^2 - 28x = 0 \Leftrightarrow x = 0, \ x = \frac{28}{9} \Rightarrow$
$\underline{\underline{x = \frac{28}{9}}}$ ty $x = 0$ kastas ty då ju $x = 0$ ger $3 \cdot 0 - 4 = 2\sqrt{4+0} \Leftrightarrow -2 = 2 > 0$

4) $\begin{cases} x^2 + y^2 = 2x + 3y + 4 \\ x^2 + y^2 = 5x + 6y + 7 \end{cases} \Leftrightarrow \begin{cases} x^2 + y^2 = 2x + 3y + 4 \\ 5x + 6y + 7 = 2x + 3y + 4 \end{cases} \Leftrightarrow \begin{cases} x^2 + y^2 = 2x + 3y + 4 \\ y = -(x+1) \end{cases}$

så $x^2 + x^2 + 2x + 1 = 2x - 3(x+1) + 4 \Leftrightarrow 2x^2 + 3x = 0 \Leftrightarrow (x_1, y_1) = (0, -1), \ (x_2, y_2) = \left(-\frac{3}{2}, \frac{1}{2}\right)$

5) $\frac{1}{x+1} < \frac{x}{x+2} \Leftrightarrow \frac{1}{x+1} - \frac{x}{x+2} < 0 \Leftrightarrow \frac{x+2-x^2-x}{(x+1)(x+2)} < 0 \Leftrightarrow \frac{-(x-\sqrt{2})(x+\sqrt{2})}{(x+1)(x+2)} < 0$

Teckenstudie:

		-2		$-\sqrt{2}$		-1		$\sqrt{2}$	
$x-\sqrt{2}$	$-$		$-$		$-$		$-$	0	$+$
$x+\sqrt{2}$	$-$		$-$	0	$+$		$+$		$+$
$x+1$	$-$		$-$		$-$	0	$+$		$+$
$x+2$	$-$	0	$+$		$+$		$+$		$+$
$\frac{-(x-\sqrt{2})(x+\sqrt{2})}{(x+1)(x+2)}$	$-$	∤	$+$	0	$-$	∤	$+$	0	$-$

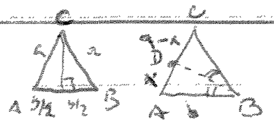
$\Rightarrow \frac{1}{x+1} < \frac{x}{x+2} \Leftrightarrow x < -2, \ -\sqrt{2} < x < -1, \ x > \sqrt{2}$

$\frac{x}{x+2} < \frac{x+1}{x+3} \Leftrightarrow \frac{x}{x+2} - \frac{x+1}{x+3} < 0 \Leftrightarrow \frac{-2}{(x+2)(x+3)} < 0$

		-3		-2	
$x+2$	$-$		$-$	0	$+$
$x+3$	$-$	0	$+$		$+$
$\frac{-2}{(x+2)(x+3)}$	$-$	∤	$+$	∤	$-$

$\Rightarrow \frac{x}{x+2} < \frac{x+1}{x+3} \Leftrightarrow x < -3$ eller $x > -2$

$\therefore$ Båda olikheterna gäller då $x < -3, \ -\sqrt{2} < x < -1, \ x > \sqrt{2}$

6) 

$\angle A = \angle B = \frac{180-36}{2} = 72^\circ$ $\angle ABD = \frac{1}{2} 72^\circ = 36^\circ \Rightarrow$ ABD likformig med ACB
Antag $|AD| = x$. Då är $|BD| = b$. Likformighet för ABD och ACB ger
$\frac{a}{b} = \frac{b}{x} \Rightarrow x = \frac{b^2}{a}$ Vidare är triangeln DBC likbent ty $\angle DBC = \angle DCB = 36^\circ$. Alltså är b
$= |BD| = |DC| = a - x \Rightarrow b = a - \frac{b^2}{a} \Leftrightarrow b^2 + ab - a^2 = 0 \Leftrightarrow b = -\frac{a}{2} \pm \sqrt{\frac{a^2}{4} + a^2} = \frac{(\pm\sqrt{5} - 1)a}{2}$
men då $b > 0$ gäller $b = a\frac{(\sqrt{5}-1)}{2}$ så $\sin 18^\circ = \frac{b}{2a} = \underline{\underline{\frac{\sqrt{5}-1}{4}}}$