Lösningar till Introduktionskurs Matematik GU 27/8-11 (1)

1a) $k = \frac{y_1 - y_2}{x_1 - x_2} = \frac{1-6}{4-2} = \frac{-5}{2}$ (= Riktningskoeff.)

Räta linjen $y - 1 = -\frac{5}{2}(x-4) \Leftrightarrow \boxed{5x + 2y = 22}$ (Svar:)

b) $\begin{cases} 2x+3y=4 \\ 5x+6y=7 \end{cases} \underset{(L_2 - 2L_1)}{\Leftrightarrow} \begin{cases} 2x+3y=4 \\ x+0=-1 \end{cases} \Leftrightarrow \boxed{\begin{cases} x=-1 \\ y=2 \end{cases}}$ (Svar:)

c) $2\ln x + 3\ln 4 = 0 \Leftrightarrow 2\ln x + 3\cdot 2\ln 2 = 0 \Leftrightarrow$

$\ln x = -3\ln 2 = \ln\left(\frac{1}{2}\right)^3 \Leftrightarrow \boxed{x = \frac{1}{8}}$ (Svar:)

d) $|2x+3| < 4 \Leftrightarrow -4 < 2x+3 < 4 \Leftrightarrow$

$-7 < 2x < 1 \Leftrightarrow \boxed{-\frac{7}{2} < x < \frac{1}{2}}$ (Svar:)

2a) $2^{x+2} - 3\cdot 2^{x+1} + 5\cdot 2^x = 1 \Leftrightarrow 4\cdot 2^x - 6\cdot 2^x + 5\cdot 2^x = 1$

$\Leftrightarrow 3\cdot 2^x = 1 \Leftrightarrow 2^x = \frac{1}{3} \Leftrightarrow x\cdot\ln 2 = \ln\left(\frac{1}{3}\right) = -\ln 3$

$\Leftrightarrow \boxed{x = -\frac{\ln 3}{\ln 2}}$ (Svar:)

b) $\cos(2x) = \sin\frac{2\pi}{5} = \cos\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) = \cos\left(\frac{5\pi - 4\pi}{10}\right) = \cos\frac{\pi}{10}$

$\Leftrightarrow 2x = \pm\frac{\pi}{10} + 2n\pi \Leftrightarrow x = \pm\frac{\pi}{20} + n\pi$, (där n = heltal)

∴ $n=0$ ger $x_1 = \frac{\pi}{20}$, $n=1$ ger $x_2 = \frac{21\pi}{20}$, $n=1$ ger $x_3 = \frac{19\pi}{20}$

och $n=2$ ger $x_4 = \frac{39\pi}{20}$. (Svar: $\boxed{\frac{\pi}{20}, \frac{19\pi}{20}, \frac{21\pi}{20}, \frac{39\pi}{20}}$

(om, $0 < x < 2\pi$)

3a) Bilda $f(x) = \frac{1}{x} - \frac{1}{x+1} \equiv \frac{x+1-x}{x(x+1)} \equiv \frac{1}{x(x+1)} > 0$ (?) 2

Teckentabell:

x	$-\infty$	$<$	-1	$<$	0	$<$	$+\infty$
$f(x)$	0	$+$	$\pm\infty$	$-$	$\pm\infty$	$+$	0

Svar: Olikheten gälles för $x<-1$ och för $x>0$

b) $(x+1)(x+2)(x+3)(x+4) = (x-1)(x-2)(x-3)(x-4) \iff$

$(x^2+3x+2)(x^2+7x+12) = (x^2-3x+2)(x^2-7x+12) \iff$

$x^4+10x^3+35x^2+50x+24 = x^4-10x^3+35x^2-50x+24$

$\iff 20x^3+100x=0 \iff x(x^2+5)=0 \iff \boxed{\begin{array}{l} x_1=0 \\ x_{2,3}=\pm i\sqrt{5} \end{array}}$

(Svar:)

4a) $VL = \sin(\alpha+\beta)\sin(\alpha-\beta) \equiv$

$\equiv (\sin\alpha\cos\beta+\cos\alpha\sin\beta)(\sin\alpha\cos\beta-\cos\alpha\sin\beta) \equiv$

$\equiv \sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta \equiv$

$\equiv \sin^2\alpha(1-\sin^2\beta) - (1-\sin^2\alpha)\sin^2\beta \equiv$

$\equiv \sin^2\alpha - \sin^2\alpha\sin^2\beta - \sin^2\beta + \sin^2\alpha\sin^2\beta \equiv$

$\equiv \sin^2\alpha - \sin^2\beta \equiv (\sin\alpha+\sin\beta)(\sin\alpha-\sin\beta) \equiv HL$

4b)

$$\begin{array}{l|l} 2x^3+3x^2+4x+5 & 3x^2+2x+1 \\ \hline -\left(2x^3+\frac{4x^2}{3}+\frac{2x}{3}\right) & \frac{2x}{3}+\frac{5}{9} = K(x) \\ \frac{5x^2}{3}+\frac{10x}{3}+5 & \\ -\left(\frac{5x^2}{3}+\frac{10x}{9}+\frac{5}{9}\right) & \\ \frac{20x}{9}+\frac{40}{9} = R(x) & \end{array}$$

(Svar:) $\boxed{K(x) = \frac{6x+5}{9} \quad R(x) = \frac{20x+40}{9}}$

5)

$$\begin{cases} (x-1)^2+(y-2)^2 = 3^2 \\ (x-4)^2+(y-5)^2 = 6^2 \end{cases} \Leftrightarrow \begin{cases} x^2-2x+1+y^2-4y+4 = 9 \\ x^2-8x+16+y^2-10y+25 = 36 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2+y^2 = 2x+4y+4 \\ x^2+y^2 = 8x+10y-5 \end{cases} \quad \therefore\ 2x+4y+4 = 8x+10y-5$$

$$\Leftrightarrow 6x+6y = 9 \Leftrightarrow y = -x+\frac{3}{2} \text{ (insättes i ekv. 1)}$$

$$\therefore\ x^2+x^2-3x+\frac{9}{4} = 2x-4x+6+4 \Leftrightarrow$$

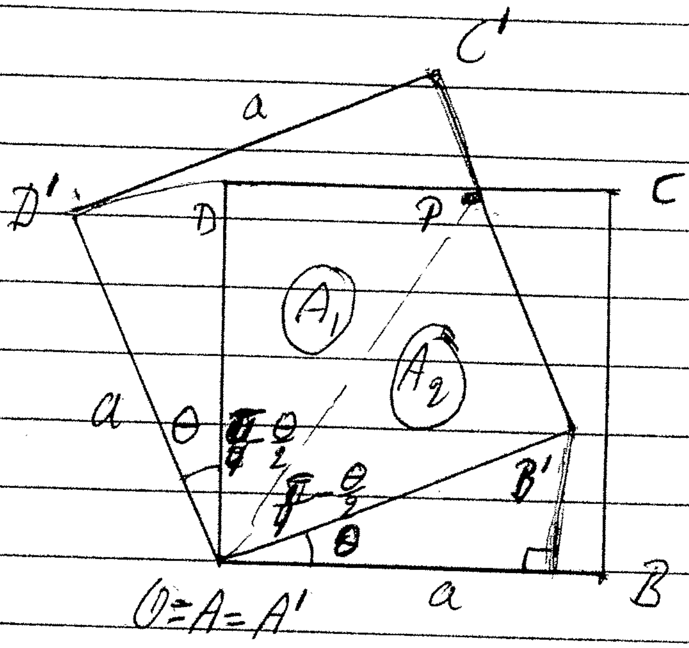
$$2x^2-x-\frac{31}{4} = 0 \Leftrightarrow x^2-\frac{x}{2}-\frac{31}{8} = 0$$

ger $x_{1,2} = \frac{1}{4} \pm \sqrt{\frac{1}{16}+\frac{31}{8}} = \frac{1\pm\sqrt{63}}{4} = \frac{1\pm3\sqrt{7}}{4}$

och $y_{1,2} = -x+\frac{3}{2} = \frac{7\mp3\sqrt{7}}{4}$

(Svar:) $\boxed{(x,y) = \frac{1}{4}\left(1\pm3\sqrt{7},\ 7\mp3\sqrt{7}\right)}$

6) Kvadraten ABCD vrides vinkeln θ, varvid fås kvadraten $A'B'C'D'$, där $A'=A=O$, (se figur).



Av symmetriskäl är $A_1 = A_2$,

och linjen OP är $y = kx$, där $k = \tan\varphi$

med $\varphi = \frac{\pi}{4} - \frac{\theta}{2} + \theta = \frac{\pi}{4} + \frac{\theta}{2} = \frac{1}{2}\left(\frac{\pi}{2} + \theta\right)$

$y = a$ ger $x_1 = \frac{a}{k} = \frac{a}{\tan\varphi} = a \cdot \frac{\cos\varphi}{\sin\varphi} =$

$= a \frac{2\sin\varphi\cos\varphi}{2\sin^2\varphi} = a \cdot \frac{\sin 2\varphi}{(1-\cos 2\varphi)} =$

$= a \frac{\sin\left(\frac{\pi}{2}+\theta\right)}{\left(1-\cos\left(\frac{\pi}{2}+\theta\right)\right)} = a \cdot \frac{\cos\theta}{(1+\sin\theta)}$

dvs $A_2 = A_1 = \frac{a}{2} \cdot a \frac{\cos\theta}{(1+\sin\theta)}$

(Svar:) $A = \boxed{a^2 \cdot \frac{\cos\theta}{(1+\sin\theta)}}$, $0 \le \theta \le \frac{\pi}{2}$. /PP