Hjälpmedel: Inga, inte ens typgodkänd räknedosa

Matematik
BB

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MMG 200 (Matematik 1, linjär algebra)

Tentamen den 18/12 2009, 8.30-13.30

1. Compute the determinant

$$\begin{vmatrix} k & k & k \\ -3 & 8 & -4 \\ 2 & -3 & 2 \end{vmatrix}$$

2. Determine all eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} -11 & -30 \\ 6 & 16 \end{pmatrix}$$

   (3 p)

3. a) Find all solutions to the system

$$\begin{aligned} x_1 + 2x_2 + 5x_3 - 3x_4 &= 0 \\ 3x_2 + x_3 + 2x_4 &= 0 \\ x_1 - x_2 + 4x_3 - 5x_4 &= 0 \\ x_1 + 5x_2 + 6x_3 - x_4 &= 0 \end{aligned}$$

   b) Find a basis for the space of solutions. (3p)

4. Let $\mathbf{u} = (1, 1, 1)$ and $\mathbf{v} = (1, -1, 0)$. Find a vector $\mathbf{w}$ which is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$. Write $(1, 0, 0)$ as a linear combination of $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$.

   (3p)

5. A linear transformation T of the plane maps the vector $\mathbf{e}_1$ to $\mathbf{e}_2$ and $\mathbf{e}_2$ to $-2\mathbf{e}_1 + 3\mathbf{e}_2$. (I e $T(\mathbf{e}_1) = \mathbf{e}_2$ and $T(\mathbf{e}_2) = -2\mathbf{e}_1 + 3\mathbf{e}_2$, where $\mathbf{e}_1$ and $\mathbf{e}_2$ is the standard basis in the plane.)

   a) Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^2$.

   b) Is there any vector $\mathbf{x}$ such that $T(\mathbf{x}) = \mathbf{x}$?

   (3p)

6. The following data points in the (x, y)-plane are obtained in an experiment:

$$(-1, -2), (0, 0), (1, 1), (3, 2).$$

a) Find the line $y = \beta_0 + \beta_1 x$ that fits the observed data best in the least square sense.

b) Give an inspired guess what the outcome of the experiment $(= y)$ might be for $x = 5$.

(3p)

7. A sequence of numbers is defined so that $x_0 = 1$, $x_1 = 1$, $x_2 = x_0 + x_1$, ... and $x_{n+1} = x_n + x_{n-1}$ if $n = 2, 3...$ (this is called the Fibonacci sequence). Define a sequence of vectors in the plane by $\mathbf{v}_n = (x_{n+1}, x_n)$.

a) Check that $\mathbf{v}_0 = (1, 1)$ and $\mathbf{v}_n = M\mathbf{v}_{n-1}$, where M is

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

b) Use a) to compute $\mathbf{v_n}$ for all n. How does x_n grow for large n?

(4p)

8. Let A be a (3×3) matrix such that the sum of the elements in any row is equal to 0. Show that the determinant of A is equal to 0.

(3p)

All vectors should be read as column vectors, although I have written them as row vectors for typographical reasons.

Good luck!