Lösningar till MMG300: 2 17 08

4) Greens formel ger $\int_\gamma y^3\,dx + (x^3+6xy^2)\,dx =$

$$= \iint_D (3x^2+6y^2) - 3y^2\,dxdy = 3\iint_D (x^2+y^2)\,dxdy =$$

$$= \text{polära koordinater} = 3\int_0^{\pi/4}\left(\int_0^1 r^2\cdot r\,dr\right)d\theta =$$

$$= \frac{3\pi}{4}\left[\frac{r^4}{4}\right]_0^1 = \frac{3\pi}{16}$$

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5) Låt D vara området i planet som ges av $x^2+y^2\le 1$.
Om vi använder x,y som parametrar får vi

$$\text{Arean} = \iint_D \sqrt{1+(z'_x)^2+(z'_y)^2}\,dxdy = \iint_D \sqrt{1+(2x+y)^2+(x-2y)^2}\,dxdy =$$

$$= \iint_D \sqrt{1+5(x^2+y^2)}\,dxdy = \{\text{polära koordinater}\} =$$

$$= \int_0^{2\pi}\left(\int_0^1 \sqrt{1+5r^2}\,r\,dr\right)d\theta = \frac{2\pi}{15}\left[(1+5r^2)^{3/2}\right]_0^1 = \frac{2\pi(6\sqrt{6}-1)}{15}$$

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6) Byt till cylindriska koordinater, $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$

Vi får området $\begin{cases} 0\le r\le z \\ 0\le\theta\le 2\pi \\ 0\le z\le 1 \end{cases}$ och

$$\int_0^{2\pi}\left(\int_0^1\left(\frac{\sin z}{z}\int_0^z r\,dr\right)dz\right)d\theta = 2\pi\int_0^1 \frac{\sin z}{z}\left[\frac{r^2}{2}\right]_0^z =$$

$$= \pi\int_0^1 z\sin z\,dz = \pi\left[-z\cos z\right]_0^1 + \pi\int_0^1 \cos z\,dz =$$

$$= \pi\left(-\cos 1 + [\sin z]_0^1\right) = \pi(\sin 1 - \cos 1)$$

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7) $\frac{nx}{n+x} = \frac{x}{1+\frac{x}{n}} \to x$, punktvis, Likformigt?

$$\left| x - \frac{nx}{n+x}\right| = \left|\frac{nx+x^2-nx}{n+x}\right| = \left|\frac{x^2}{n+x}\right| \le \frac{1}{n} \text{ då } 0\le x\le 1$$

$\frac{1}{n}\to 0$ oberoende av x. Konvergensen likformig.

Därmed $\lim_{n\to\infty}\int_0^1 \frac{nx}{n+x}\,dx = \int_0^1 \lim_{n\to\infty}\frac{nx}{n+x}\,dx = \int_0^1 x\,dx = \frac{1}{2}$

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8)

$$y = \sum_0^\infty c_k x^k$$

$$y' = \sum_1^\infty c_k k x^{k-1} = \sum_0^\infty c_{k+1}(k+1)x^k$$

$$y'' = \sum_0^\infty c_{k+1}\,k(k+1)\,x^{k-1},$$

$$xy'' = \sum_0^\infty c_{k+1}\,k(k+1)\,x^k$$

Insättning i ekvation ger

$$\sum_0^\infty \left(c_{k+1}k(k+1) + c_{k+1}(k+1) - c_k\right)x^k = 0$$

Dvs $c_{k+1}[k(k+1)+k+1] = c_k$ eller

$$c_{k+1} = \frac{1}{(k+1)^2}c_k$$

$y(0)=1 \Rightarrow c_0 = 1$ och

$$c_1 = \frac{1}{1^2}c_0 = 1,\quad c_2 = \frac{1}{2^2},\quad c_3 = \frac{1}{3^2}\cdot\frac{1}{2^2},\ \dots$$

$$\dots\quad c_k = \frac{1}{k^2(k-1)^2\cdots 2^2\cdot 1} = \frac{1}{(k!)^2}$$

$$y = \sum_0^\infty \frac{x^k}{(k!)^2}$$