1, 2, 3 + 5c PB.

4. a) Kpt. b) Beg., ej sluten c) Kpt. d) Öppen, ej beg.

5. $f(x,y) = ye^x - xy^2$, $D = \{(x,y) : \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le 2 \end{matrix}\}$

i) $0 < x < 1,\ 0 < y < 2$: $\nabla f(x,y) = (ye^x - y^2,\ e^x - 2xy) = (0,0)$

$\Rightarrow \begin{cases} y = e^x \\ 2x = 1 \end{cases} \Rightarrow \begin{cases} x = 1/2 \\ y = e^{1/2} \end{cases}$

$(Hf)(1/2, e^{1/2}) = \begin{pmatrix} ye^x & e^x - 2y \\ e^x - 2y & -2x \end{pmatrix}_{(x,y)=(1/2,e^{1/2})} = \begin{pmatrix} e^1 & -e^{1/2} \\ -e^{1/2} & -2\cdot\frac{1}{2} \end{pmatrix}$

$\Rightarrow$ $(1/2, e^{1/2})$ lokalt max, $f(1/2, e^{1/2}) = \frac{1}{2}e^1 < 2$ ty sg. def.

ii) $x = 0 \quad y = t$, $0 \le t \le 2$

$g(t) = f(0,t) = t$ Min = 0 $(t=0)$ Max = 2 $(t=2)$

iii) $x = t \quad y = 0$, $0 \le t \le 1$

$g(t) = f(t,0) = 0$ Min = Max = 0

iv) $x = t \quad y = 2$, $0 \le t \le 1$

$g(t) = 2e^t - 4t$ $g'(t) = 2e^t - 4 = 0 \Rightarrow e^{t_0} = 1/2$

$\Rightarrow t_0 < 0$ (ej tillåtet)

$g(0) = 2$ $g(1) = 2e^1 - 4 < 2$.

v) $x = 1 \quad y = t$, $0 \le t \le 2$

$g(t) = f(1,t) = te^1 - t^2$ $g'(t) = e^1 - 2t = 0$

$\Rightarrow t = e^{1/2}$

$g(0) = 0$ $g(2) = 2e^1 - 4 > 0$.

$g(e^{1/2}) = \frac{1}{2}e^2 - \frac{1}{4}e^2 = \frac{1}{4}e^2 < 2$

Max = 2
Min = 0

6. $M_1 = \{(x,y,z) : z = f(x,y)\}$, $M_2 = \{(x,y,z) : z = (2+f(x,y))^2\}$

$(0,0,0) \in M_1$, $(0,0,4) \in M_2$

Tangentplan: $\underbrace{\frac{\partial f}{\partial x}(0,0)}_{a} \cdot x + \underbrace{\frac{\partial f}{\partial y}(0,0)}_{b} \cdot y \underbrace{-}_{c} z = 0$

Tangentplan:

$$2\frac{\partial f}{\partial x}(0,0)\,(2+\underbrace{f(0,0)}_{0})\cdot x + 2\frac{\partial f}{\partial y}(0,0)\,(2+\underbrace{f(0,0)}_{0})\,y - (z-4) = 0$$

$$2a \cdot 2x + 2b \cdot 2y - (z-4) = 0$$

$$\boxed{4a \cdot x + 4b \cdot y - z = -4}$$

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7. $f(x,y) = \frac{1}{\sqrt{xy}}\, g\left(\frac{x}{y}\right)$;

$$f'_x = \frac{1}{\sqrt{y}}\left[(-1/2)\cdot x^{-3/2}\cdot g\left(\frac{x}{y}\right) + \frac{x^{-1/2}}{y}\cdot g'\left(\frac{x}{y}\right)\right]$$

$$f'_y = \frac{1}{\sqrt{x}}\left[(-1/2)\,y^{-3/2}\cdot g\left(\frac{x}{y}\right) + y^{-1/2}\cdot x(-y^{-2})\cdot g'\left(\frac{x}{y}\right)\right]$$

$$x f'_x + y f'_y + a\cdot f = -\frac{1}{2\sqrt{xy}}\, g\left(\frac{x}{y}\right) + \frac{x^{1/2}}{y^{3/2}}\, g'\left(\frac{x}{y}\right)$$

$$- \frac{1}{2\sqrt{xy}}\, g\left(\frac{x}{y}\right) - \frac{x^{1/2}}{y^{3/2}}\, g'\left(\frac{x}{y}\right)$$

$$+ \frac{a}{\sqrt{xy}}\, g\left(\frac{x}{y}\right)$$

$$= \frac{(a-1)}{\sqrt{xy}}\cdot g\left(\frac{x}{y}\right) = 0 \quad \text{om } \boxed{a=1}$$

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8. $$\lim_{r\to 0+} \frac{f(r\cos\theta, r\sin\theta) - \overbrace{f(0,0)}^{0}}{r}$$

$$= \lim_{r\to 0+} \frac{2r^4\cos^4\theta + 3r^4\sin^4\theta + r^3\cos^3\theta}{r^3} = \cos^3\theta$$

Om f diff, $f(0,0) = 0$, så gäller

$$f(r\cos\theta, r\sin\theta) = (a\cos\theta + b\sin\theta)\cdot r + O(r^2)$$

för ngr. konstanter a, b — Motsägelse!