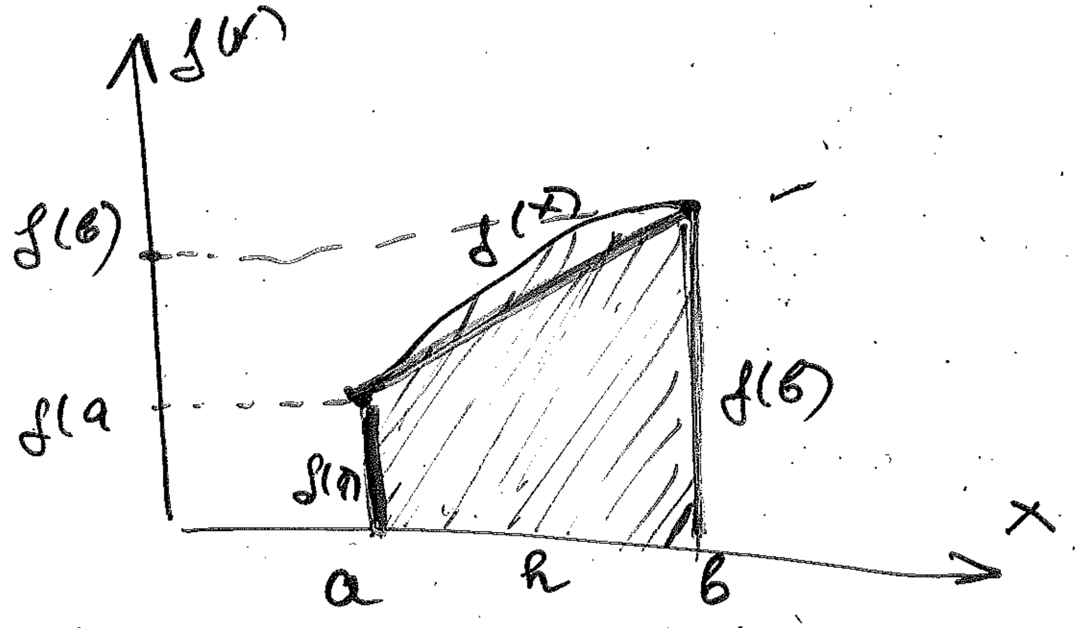
Trapetsmetoden

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a)+f(b)\right),$$

$$h = b-a$$



$$\int_a^b f(x)\,dx = \sum_{i=1}^{N-1}\int_{x_i}^{x_{i+1}} f(x)\,dx \approx \frac{h}{2}\Big[f(x_1) + \underbrace{f(x_2) + (f(x_2)}_{2\ \text{gånger}} + \underbrace{f(x_3)) + (f(x_3)}_{2\ \text{gånger}} + f(x_4)) + \dots + (f(x_{N-1}) + f(x_N))\Big] =$$

$$= h\left[\frac{f(x_1)}{2} + f(x_2) + f(x_3) \dots + f(x_{N-1}) + \frac{f(x_N)}{2}\right].$$

Interpolationsfelet: $\leq M$

$$\underbrace{f(t)}_{\substack{\text{exakt}\\ \text{funktion}}} - \underbrace{p(t)}_{\substack{\text{interpol.}\\ \text{polynom}}} = \frac{f^{(n)}(Q)}{n!}(t-t_1)(t-t_2)\dots(t-t_n)$$

$Q \in (t, t_1, \dots, t_n)$.

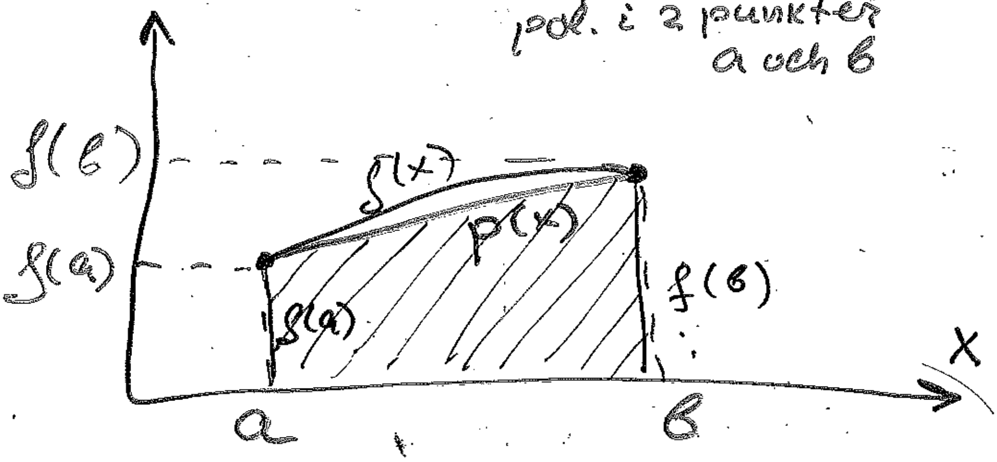
Antar att $|f^{(n)}(Q)| \leq M \quad \forall Q$, då

$$|f(t) - p(t)| \leq \frac{M}{n!}(t-t_1)\cdots(t-t_n)$$

För 2 punkter a och b:

$$f(x) - \underbrace{p(x)}_{\substack{\text{interp.}\\ \text{pol. i 2 punkter}\\ a \text{ och } b}} = \frac{f''(Q)}{2}(x-a)(x-b)$$

$Q \in (a, b)$



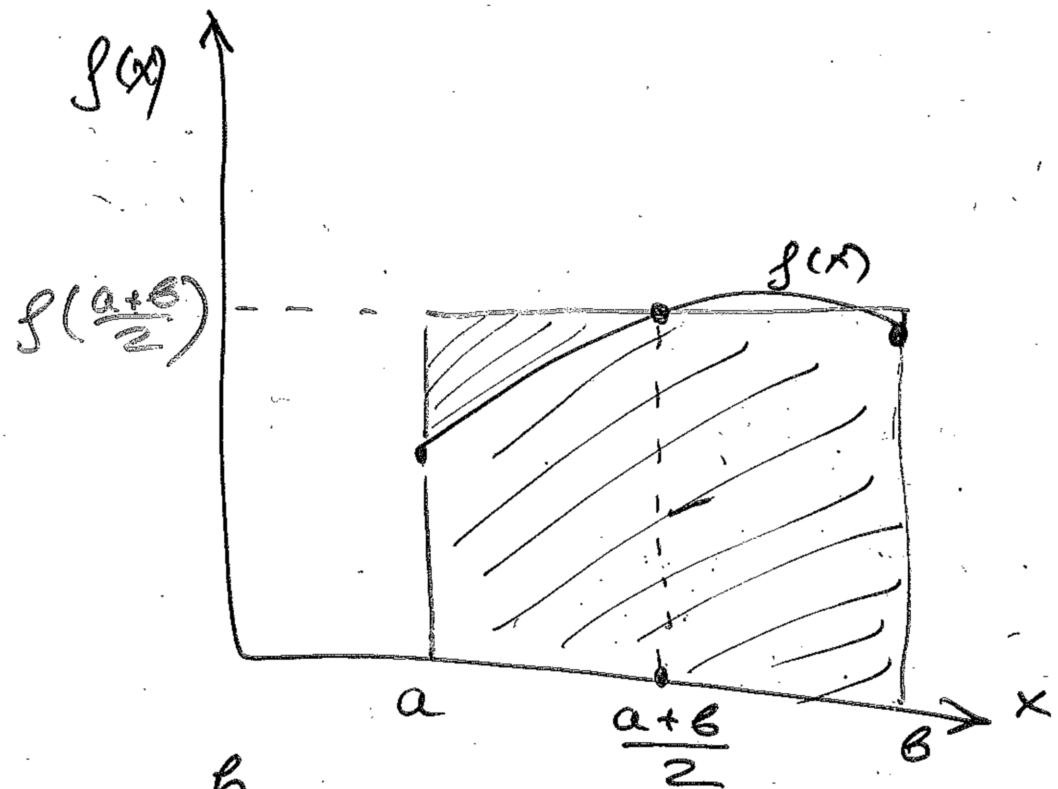
int. pol. i 2 punkter $\rightarrow p(x) = f(a) + (x-a)\dfrac{f(b)-f(a)}{b-a}$

Integral: $\displaystyle\int_a^b f(x)\,dx \approx \int_a^b p(x)\,dx =$

$$= \int_a^b \left[f(a) + (x-a)\frac{f(b)-f(a)}{b-a}\right]dx = \underbrace{\frac{f(a)+f(b)}{2}\cdot h}_{\text{trapetsmetoden}}$$

$h = b - a$

Rektangelmetoden



$$\int_a^b f(x)\,dx \approx (b-a)\cdot f\left(\frac{a+b}{2}\right)$$

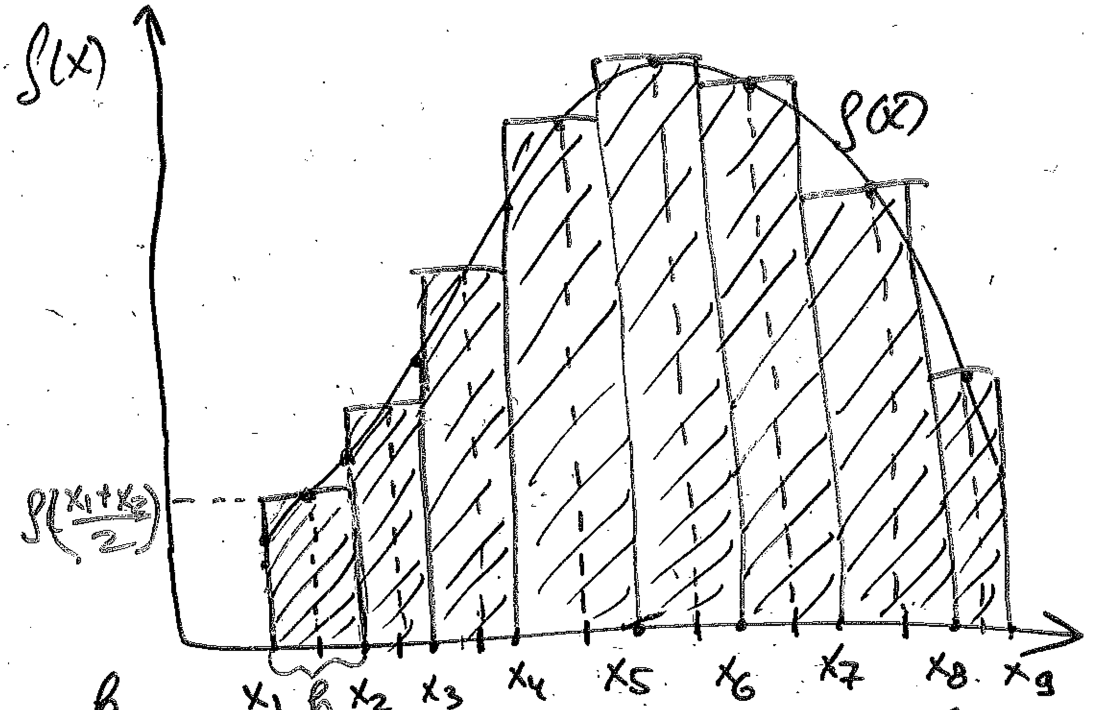
$$\int_a^b f(x)\,dx \approx \frac{h}{2}\Big[\big(f(x_1)+f(x_2)\big)+\big(f(x_2)+f(x_3)\big)+$$

$$+\dots+\big(f(x_{n-1})+f(x_n)\big)\Big] =$$

$$= h\left[\frac{f(x_1)}{2}+\sum_{i=2}^{n-1} f(x_i)+\frac{f(x_n)}{2}\right]$$

$$h=\frac{b-a}{n-1}$$

Rektangelmetoden



$$\int_a^b f(x)\,dx \approx (b-a)\, f\left(\frac{a+b}{2}\right)$$

$$\int_a^b f(x)\,dx \approx h\left[f\left(\frac{x_1+x_2}{2}\right)+f\left(\frac{x_2+x_3}{2}\right)+\dots f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$$

$$h = x_n - x_{n-1}, \quad h = \frac{b-a}{n-1}$$

Trapetsmetoden

$$\int_a^b f(x)\,dx \approx \frac{f(a)+f(b)}{2}\cdot h; \quad \int_a^b f(x)\,dx \approx h\left[\frac{f(x_1)}{2}+\dots+\sum_{i=2}^{n-1} f(x_i)+\frac{f(x_n)}{2}\right]$$